Strategic Informed Trades, Diversification, and Expected Returns*

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Abstract

We examine how imperfect competition impacts expected returns in the large economy limit of a noisy rational expectation equilibrium. In our model, both an informed and uninformed traders consider the impact of their demands on price. Similar to the case of competitive informed traders, we show that factor loadings (betas) explain all cross-sectional differences in expected returns. Private information creates two forces that have opposite effects on expected returns. On the one hand, prices partially reveal private information about systematic risks, which reduces factor risk premiums. On the other hand, privately informed traders reduce their absorption of systematic risk, which increases factor risk premiums. In a setting with a monopolist privately informed trader, the latter effect dominates, which contrasts to settings with price-taking informed traders. Moreover, a reduction in disguise for informed trades afforded by noise trade volatility leads to greater price informativeness, but also higher expected returns, suggesting a new avenue for empirical examination of the pricing effects of asymmetric information. In particular, noise trades in diversified portfolios reduce price informativeness about systematic risks, but facilitate large traders' bearing of systematic risks and reduce expected returns.

1 Introduction

Accounting research has devoted considerable attention to the effects of asymmetric information on expected returns. Prior theoretical studies show that, in large economies comprised of perfectly competitive traders who can fully diversify their holdings, asymmetric information only affects expected returns via its impact on premiums for systematic risks (Hughes et al. 2007; Lambert et al. 2007). In particular, cross-sectional differences in factor loadings (betas) explain any cross-sectional differences in expected returns.[1] In a CARA/normal setting, the premium for systematic risk depends on the average precision of traders' beliefs (Hughes et al. 2007; Lambert et al. 2012). Compared to a no-information benchmark, asymmetric information reduces expected returns by increasing the average precision of traders' beliefs. For a given set of informative signals, expected returns are higher if only a subset of traders observes them privately, because uninformed traders only observe a noisy version of private information *via* prices.

We examine whether similar results hold when modeling the economy as the limit of finite economies in which traders participate in imperfectly competitive markets, and therefore take into account the price impact of their trades. Our model can be viewed as a multi-asset extension of Kyle (1989). In order to emphasize the consequences of imperfectly competitive informed trading, we portray the informed trader as a risk-neutral monopolist who can observe prices (i.e., places limit orders as in Kyle 1989). Risk-neutrality allows the informed trader to have a non-negligible impact on prices in the large economy limit.[2] The informed trader plays

[1] When we speak of expected returns, we refer to unconditional expected returns. Privately informed traders may, and often do, expect returns in excess of the compensation for systematic risk.

[2] A risk-averse informed trader yields qualitatively similar results in a finite economy so long as risk aversion is not too great. The similarity stems from the fact that uninformed traders drive the unconditional expected returns. In the large economy limit, risk neutrality ensures sufficient risk-bearing capacity for the demands of the informed trader to have a price impact. With finite risk tolerance, an informed trader will bear only a negligible amount of systematic risk in the limiting large economy, resulting in large-economy expected returns that match an economy in

two roles in our model: she exploits private information and provides liquidity to noise traders in order to capture the systematic risk premium. Risk neutrality maximizes the informed trader's incentive for information-based trades, but also positions her to capture a portion of the systematic risk premium. In the absence of private information, the informed trader extracts half of the risk premium. Given private information, the informed trader curbs her demands to limit the revelation of her private information to uninformed traders, thereby absorbing less risk as a counter party to noise trades. It follows that uninformed traders bear greater systematic risk for which they require a higher expected return.[3]

Diversification by uninformed traders plays a key role in driving our large economy results with respect to expected returns. In the limit, uninformed traders diversify, which implies that trades in a given asset represent only a small fraction of an individual uninformed trader's portfolio, and a small fraction of overall trade in the asset.[4] This causes uninformed traders to constructively behave as price takers. At the margin, prices depend on diversified uninformed traders' demands and, therefore, include risk premiums only for non-diversifiable (systematic) risk. If, at the margin, an uninformed trader were to infer that an individual asset is mispriced, he could initiate small trades in that asset without affecting his overall risk exposure due to local risk-neutrality. Because all uninformed will draw the same inference that the asset is mispriced, all would adjust their demands. In equilibrium, perfect competition among uninformed traders would dissipate the pricing of any diversifiable risk.

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which those traders do not exist. The result that asymmetric information only impacts systematic risk premiums is unaffected in this case.

[3] Diamond and Verrecchia (1991) predict a similar phenomenon in a single-firm setting where expected future market illiquidity increases large traders' expected costs of unwinding large positions. While the settings differ, in both cases, illiquidity increases the risk that must be borne by uninformed traders and drives up expected returns.

[4] In principle, uninformed traders could have some undiversified holdings so long as they do not all overweight the same individual assets. In our model, each asset represents a small fraction of the economy so that a nontrivial fraction of uninformed traders can have an undiversified position in a given asset only if other traders take the opposite side of the trade.

We develop further insights by considering extreme cases of market liquidity. When noise trade is so volatile as to preclude uninformed traders learning from prices, the risk-neutral informed trader absorbs half of the liquidity demands as the profit maximizing share of the systematic risk premium. At the other extreme, when the variance of noise trades approaches zero, trading costs become infinitely large and the informed trader only takes infinitesimal positions, which leaves uninformed traders to absorb systematic risks associated with expected noise trades.[5] After imposing structure on the informed trader's private information and noise trades, we show that the impact of risk-sharing dominates the effect of information revealed through price, leading to a monotone relation between market liquidity and expected return; i.e., expected returns are decreasing in the variance of noise trades, even though prices become less informative. In this same setting, we also show that the introduction of private information increases expected returns because the reduction in the informed trader's absorption of systematic risk more than offsets the effect of the partial revelation of private information through prices.

We further show that noise traders' portfolios play a key role in the pricing of systematic risk. If the informed trader has private information on systematic risks, then she needs to trade in fully diversified portfolios to exploit that information. She can only do so if noise traders also trade in diversified portfolios; otherwise, uninformed traders can infer her trades, similar to zero-noise-trade-variance case.[6] As the variance of noise trades in factor portfolios increases, the informed trader's information-based demands absorb more systematic risks, causing expected returns required by uninformed traders to decrease. Even though greater noise trade variance reduces the uninformed traders' learning about systematic risks, expected returns decline. Again,

[5] We can only examine the zero noise-trade variance case with competitive uninformed investors. The market breaks down if uninformed traders consider their price impact and there is no noise in supply (Kyle 1989, p. 335).

[6] See Subrahmanyam (1991) for a model in which informed traders can trade a diversified index to bet on systematic risks. His model has all risk-neutral traders so that expected returns are always zero.

this occurs because the risk-neutral informed trader's larger positions reduce the systematic risks borne by uninformed traders and the corresponding risk premiums. This suggests that liquidity trades in diversified portfolios, such as automatic investments into index funds for retirement accounts, improve risk sharing by drawing informed traders to take larger positions that absorb systematic risk.

Recent empirical studies by Armstrong, Core, Taylor, and Verrecchia (2011) and Akins, Ng, and Verdi (2012) show a positive association between imperfect competition among informed traders and cost of capital. The theoretical basis is the prediction that when the number of informed traders is small, they trade less aggressively on their private information so as to limit the information that uninformed traders can learn from price. This lowers the average precision of information and increases the risk borne by uninformed traders, which raises cost of capital. In our model, however, expected returns depend solely on systematic risks, so that any cross-sectional variation in expected returns will vanish after controlling for factor loadings. Of course, it is also possible that uninformed investors are not well diversified, in which case firm-specific risks may be priced. This would allow for cross-sectional variations in cost of capital even after controlling for betas.[7]

Another direction for future empirical inquiries is to consider the extent of uninformed shareholders' diversification when examining the effects of information asymmetries on expected returns. For example, Faccio, Marchica, and Mura (2011) find empirical evidence that firms with shares held by investors with well diversified portfolios care less about firm-specific risk taking. Controlling for betas in such a context, we suggest consideration of numerator, or cash flow, effects as drivers of cross-sectional differences in expected return. For an example in

[7] However, this poses a conundrum in that it is unlikely that uninformed traders would irrationally under diversify and, yet, be sufficiently rational to extract information from prices.

a moral hazard setting, Gao and Verrecchia (2012) show that idiosyncratic risk increases the risk premium paid to managers as part of a performance-based compensation package (a numerator effect), while the expected return to investors (denominator effect) depends entirely on systematic risk.[8] In their setting, better firm-specific information reduces premiums required to induce managerial effort, while better economy-wide information reduces systematic risk and, hence, expected return.[9]

Relating our results to real markets, we might envision hedge fund managers as large, risk tolerant traders with private information who may profit from capturing risk premiums, as well as from exploiting an information advantage. Accordingly, better private information might translate into less risk absorption by hedge funds, leading to higher expected returns by uninformed traders who are left to cover liquidity demands of noise traders. Holding the precision of private information constant, greater market liquidity in the sense of more volatile noise trades might provide greater disguise for information-based hedge fund trades leading to higher expected returns. We are unaware of empirical inquiries addressed to such a prediction, suggesting a new avenue for future empirical research.

The remainder of the paper is organized as follows: Section 2 characterizes equilibrium expected returns in a finite economy, section 3 takes the large economy limit, section 4 extends the analysis after imposing additional structure on information and noise trades. Section 5 concludes.

[8] Also, see Ou-Yang (2005) for a related moral hazard setting. See Christensen, Feltham, and Wu (2002) for a setting in which the imposition of idiosyncratic risk on managers impacts the rate of return used in residual income compensation metrics, again resulting in numerator effects of idiosyncratic risk.

[9] In some cases, concerns over cash flow effects can provide incentives to reduce the information provided by the accounting system. For example, Caskey and Hughes (2012) show that conservative accounting measures, which provide distorted information, have beneficial cash flow effects because they mitigate asset substitution problems in levered firms. In their case, more informative (unbiased) accounting reports can reduce firm value.

2 Equilibrium in a finite economy

2.1 Equilibrium

Model setup

In this section, we derive a linear equilibrium in prices for a finite set of risky assets. As in Kyle's (1989) analysis in a single risky asset setting, we assume that both an informed and uninformed traders observe prices when choosing their demands; i.e., place limit orders. Traders allocate their demands between a risk-free asset, with return normalized to zero, and N risky assets, and share the prior belief that the vector $\boldsymbol{v}$ of the risky assets' payoffs is normally distributed with mean $\bar{\boldsymbol{v}}$ and covariance matrix $\boldsymbol{\Sigma}_v$. The supply of risky assets, net of noise trades, $\boldsymbol{x}$ is a $N \times 1$ vector of normally distributed random variables with mean $\bar{\boldsymbol{x}}$, covariance matrix $\boldsymbol{\Sigma}_x$, and independent of $\boldsymbol{v}$.

We assume that a risk-neutral informed trader has private information yielding the posterior belief that $\boldsymbol{v}$ has mean $\bar{\boldsymbol{v}}_i$ and covariance matrix $\boldsymbol{\Sigma}_i = \boldsymbol{\Sigma}_v - \boldsymbol{\Sigma}_{\bar{v}_i}$. We further assume that $\bar{\boldsymbol{v}}_i$ is joint normally distributed with $\boldsymbol{v}$ and $\boldsymbol{x}$, and is uncorrelated with $\boldsymbol{x}$. The combination of risk neutrality and monopoly on private information implies a strong incentive for distorting trades to exploit an information advantage.[10] Risk neutrality also implies a comparative advantage and related incentive to capture the risk premium. As we will demonstrate, these incentives create a tension that plays an important role in determining equilibrium expected return. The informed trader places demands $y_n, n = 1, \ldots, N$ to maximize her expected payoff:

$$\mathrm{E}_i\left[\sum_{n=1}^{N} y_n (v_n - p)\right] = \mathrm{E}_i\left[\boldsymbol{y}'(\boldsymbol{v} - \boldsymbol{p})\right] = \boldsymbol{y}'(\bar{\boldsymbol{v}}_i - \mathrm{E}_i[\boldsymbol{p}]), \tag{1}$$

[10] As we mention in the introduction, risk neutrality is sufficient to ensure a price impact of the informed trader's demands in the large economy limit. We obtain similar results in a setting with multiple risk-neutral traders who place market orders, as in Kyle (1985), so long as there are more uninformed traders than informed traders. A linear equilibrium with multiple, risk-neutral informed traders who possess homogeneous private information does not exist (See, e.g., Back, Cao, and Willard 2000).

where $\boldsymbol{y}'\boldsymbol{p}$ is the opportunity cost of holding those shares.

There are M uninformed traders with CARA utility and risk-aversion A. In deriving a linear equilibrium, we assume it is common knowledge that the uninformed traders conjecture that the informed trader follows a linear strategy of the form

$$\boldsymbol{y} = \boldsymbol{q}_0 + \boldsymbol{Q}_v(\overline{\boldsymbol{v}}_i - \overline{\boldsymbol{v}}) - \boldsymbol{Q}_p\boldsymbol{p}, \tag{2}$$

and that the informed trader and uninformed traders $m' \neq m$ conjecture that trader m follows a linear strategy of the form

$$\boldsymbol{d}_m = \boldsymbol{c}_0 - \boldsymbol{C}_p\boldsymbol{p}. \tag{3}$$

These conjectures imply that uninformed trader m's posterior belief about $\boldsymbol{v}$ is normally distributed, allowing us to write the objective function in terms of certainty equivalents:

$$\max_{\boldsymbol{d}_m} \boldsymbol{d}_m'(\mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}] - \boldsymbol{p}) - \tfrac{1}{2}A\boldsymbol{d}_m'\boldsymbol{\Sigma}_{v|p}\boldsymbol{d}_m. \tag{4}$$

The market clearing condition is:

$$\boldsymbol{y} + \sum_{m=1}^{M}\boldsymbol{d}_m = \boldsymbol{x}. \tag{5}$$

Trading strategies

Conjectures depicted by equations (2) and (3) along with the market clearing condition (5) allow us to treat the informed trader and uninformed trader m as optimizing against the following residual supply curves, respectively:

$$\boldsymbol{p} = \boldsymbol{\mu}_i + \boldsymbol{\Lambda}_i\boldsymbol{y}, \qquad \boldsymbol{p} = \boldsymbol{\mu}_u + \boldsymbol{\Lambda}_u\boldsymbol{d}_m, \tag{6}$$

where $\boldsymbol{\mu}_i$ ($\boldsymbol{\mu}_u$) does not depend on $\boldsymbol{y}$ ($\boldsymbol{d}_m$). Matrices $\boldsymbol{\Lambda}_i$ and $\boldsymbol{\Lambda}_u$ reflect the sensitivities of prices to informed and uninformed traders demands, respectively. Substituting the informed trader's residual supply curve (6) into the objective function (1) yields the following two equivalent expressions of the informed trader's first-order condition, $\boldsymbol{0} = \overline{\boldsymbol{v}}_i - \boldsymbol{\mu}_i - (\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')\boldsymbol{y}$:

$$\boldsymbol{y} = (\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}(\overline{\boldsymbol{v}}_i - \boldsymbol{\mu}_i), \quad \text{and} \quad \boldsymbol{y} = (\boldsymbol{\Lambda}_i')^{-1}(\overline{\boldsymbol{v}}_i - \boldsymbol{p}), \tag{7}$$

with the second-order condition that $\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i'$ is positive definite. The term $\overline{\boldsymbol{v}}_i - \boldsymbol{p}$ reflects the informed trader's expected per share profit, and $\boldsymbol{\Lambda}_i'\boldsymbol{y}$ reflects her anticipation of the impact of her demands on price. Her trade given by (7) equates her marginal profit $\overline{\boldsymbol{v}}_i - \boldsymbol{p}$ per share to the adverse impact $\boldsymbol{\Lambda}_i'\boldsymbol{y}$ of her demands on price; i.e., $\boldsymbol{\Lambda}_i'\boldsymbol{y} = (\overline{\boldsymbol{v}}_i - \boldsymbol{p})$.

When inferring information from price, uninformed trader m can apply the conjectured uninformed strategy given by (3) and informed strategy given by (2) to the market clearing condition given by (5) to derive a noisy signal $\boldsymbol{s}$ of the informed trader's information $\overline{\boldsymbol{v}}_i - \overline{\boldsymbol{v}}$:

$$\begin{aligned} &\underbrace{\boldsymbol{q}_0 + \boldsymbol{Q}_v(\overline{\boldsymbol{v}}_i - \overline{\boldsymbol{v}}) - \boldsymbol{Q}_p\boldsymbol{p}}_{y} + \boldsymbol{d}_m + (M-1)(\boldsymbol{c}_0 - \boldsymbol{C}_p\boldsymbol{p}) = \boldsymbol{x} \\ &\qquad \Rightarrow \boldsymbol{s} = \boldsymbol{Q}_v^{-1}\left(\boldsymbol{Q}_p\boldsymbol{p} + \overline{\boldsymbol{x}} - \boldsymbol{q}_0 - \boldsymbol{d}_m - (M-1)(\boldsymbol{c}_0 - \boldsymbol{C}_p\boldsymbol{p})\right) = \overline{\boldsymbol{v}}_i - \overline{\boldsymbol{v}} - \boldsymbol{Q}_v^{-1}(\boldsymbol{x} - \overline{\boldsymbol{x}}). \end{aligned} \tag{8}$$

This yields the posterior beliefs:

$$\mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}] = \mathrm{E}[\boldsymbol{v} \mid \boldsymbol{s}] = \overline{\boldsymbol{v}} + \boldsymbol{\Sigma}_{\overline{v}_i}(\boldsymbol{\Sigma}_{\overline{v}_i} + \boldsymbol{\Sigma}_s)^{-1}\boldsymbol{s}, \quad \boldsymbol{\Sigma}_{v|p} = \boldsymbol{\Sigma}_v - \boldsymbol{\Sigma}_{\overline{v}_i}(\boldsymbol{\Sigma}_{\overline{v}_i} + \boldsymbol{\Sigma}_s)^{-1}\boldsymbol{\Sigma}_{\overline{v}_i}, \tag{9}$$

where $\boldsymbol{\Sigma}_s = \boldsymbol{Q}_v^{-1}\boldsymbol{\Sigma}_x(\boldsymbol{Q}_v^{-1})'$ and $\mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}]$ do not vary with $\boldsymbol{d}_m$. The uninformed trader m's residual demand curve given by (6) and the first-order condition given by (4) imply that $\boldsymbol{d}_m$ satisfies the following equivalent expressions:

$$\boldsymbol{d}_m = (\boldsymbol{\Lambda}_u + \boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})^{-1}(\mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}] - \boldsymbol{\mu}_u), \quad \text{and} \quad \boldsymbol{d}_m = (\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})^{-1}(\mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}] - \boldsymbol{p}), \tag{10}$$

where $\boldsymbol{p}$ depends on $\boldsymbol{d}_m$, and the second-order condition requires that the matrix $\boldsymbol{\Lambda}_u + \boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p}$ be positive definite. The uninformed traders weigh their expected profit $\mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}] - \boldsymbol{p}$ not only against their anticipation of the adverse impact of their demands $\boldsymbol{\Lambda}_u'\boldsymbol{d}_m$ on price, but, as well, the risk $A\boldsymbol{\Sigma}_{v|p}\boldsymbol{d}_m$ that they absorb, a brake on demands not present for the risk neutral informed trader.

Equilibrium

The following proposition summarizes the equilibrium up to the matrix $\boldsymbol{\Lambda}_i'$:

Proposition 1:

The equilibrium price, expected returns, informed demand, and uninformed demands are:

$$\begin{aligned}\boldsymbol{p} &= \mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}] - (\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})\boldsymbol{d}_m \\ &= \overline{\boldsymbol{v}} - \tfrac{1}{M}\left(\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p}\right)(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I} + \boldsymbol{\Sigma}_{\overline{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Lambda}_i'\overline{\boldsymbol{x}} + \boldsymbol{\Lambda}_i(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}\left(\overline{\boldsymbol{v}}_i - \overline{\boldsymbol{v}} - \boldsymbol{\Lambda}_i'(\boldsymbol{x} - \overline{\boldsymbol{x}})\right),\end{aligned} \tag{11}$$

$$\mathrm{E}[\boldsymbol{v} - \boldsymbol{p}] = \underbrace{\tfrac{1}{M}\left(\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p}\right)}_{\substack{\text{Effect of risk-aversion}\\ \text{and imperfect competition}\\ \text{among uninformed}}} \underbrace{(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I} + \boldsymbol{\Sigma}_{\overline{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Lambda}_i'}_{\substack{\text{Effect of informed trader's}\\ \text{absorption of risk associated}\\ \text{with expected noise trades}}}\overline{\boldsymbol{x}}, \tag{12}$$

$$\boldsymbol{y} = \underbrace{(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}\left(\boldsymbol{\Lambda}_i - \boldsymbol{\Sigma}_{\overline{v}_i}\boldsymbol{\Sigma}_s^{-1}\boldsymbol{\Lambda}_i'\right)\overline{\boldsymbol{x}}}_{\substack{\text{Uninformative part of demands related to}\\ \text{absorption of risk associated with}\\ \text{expected noise trades}}} + \underbrace{(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}\left(\overline{\boldsymbol{v}}_i - \overline{\boldsymbol{v}} + \boldsymbol{\Lambda}_i(\boldsymbol{x} - \overline{\boldsymbol{x}})\right)}_{\substack{\text{Informative part of demands; related to}\\ \text{private information and absorption of risk}\\ \text{associated with unexpected noise trades}}}, \tag{13}$$

$$\boldsymbol{d}_m = \tfrac{1}{M}(\boldsymbol{x} - \boldsymbol{y}) = \tfrac{1}{M}(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}\left((\boldsymbol{I} + \boldsymbol{\Sigma}_{\overline{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Lambda}_i'\overline{\boldsymbol{x}} - \left(\overline{\boldsymbol{v}}_i - \overline{\boldsymbol{v}} - \boldsymbol{\Lambda}_i'(\boldsymbol{x} - \overline{\boldsymbol{x}})\right)\right), \tag{14}$$

where $\boldsymbol{\Lambda}_u = \boldsymbol{\Lambda}_i\left(\boldsymbol{\Lambda}_i + \tfrac{M-1}{M}\boldsymbol{\Lambda}_i'\right)^{-1}\boldsymbol{\Lambda}_i'$.

As mentioned earlier, the $\boldsymbol{\Lambda}_u'$ term in (11) arises from uninformed traders anticipating the impact of their demands on prices. The $\boldsymbol{\Sigma}_{\overline{v}_i}\boldsymbol{\Sigma}_s^{-1}$ term resembles a signal ($\boldsymbol{\Sigma}_{\overline{v}_i}$) to noise ($\boldsymbol{\Sigma}_s$) ratio and reflects the informed trader's sensitivity to revealing information. *Ceteris paribus*, the informed trader becomes less aggressive when prices reveal more of her information (higher $\boldsymbol{\Sigma}_{\overline{v}_i}\boldsymbol{\Sigma}_s^{-1}$). This reduces the informed trader's absorption of expected noise trades $\overline{\boldsymbol{x}}$ and related risk. The informed trader's absorption of $\overline{\boldsymbol{x}}$ can also be written in terms of the sensitivities of prices to both informed and uninformed traders' demands; i.e.:

$$(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{\Lambda}_i - \boldsymbol{\Sigma}_{\overline{v}_i}\boldsymbol{\Sigma}_s^{-1}\boldsymbol{\Lambda}_i')\overline{\boldsymbol{x}} = (\boldsymbol{\Lambda}_i')^{-1}\left((\boldsymbol{\Lambda}_i')^{-1} + M\left(\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p}\right)^{-1}\right)^{-1}\overline{\boldsymbol{x}}, \tag{15}$$

where the absorption depends on the magnitude of informed trader's responsiveness $(\boldsymbol{\Lambda}_i')^{-1}$ to expected profits from (7), relative to the total of the informed trader's responsiveness and the uninformed traders' responsiveness $M\left(\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p}\right)^{-1}$ to expected profits from (10). Recall that the informed trader absorbs noise trades in order to capture the risk premium on such trades and that this incentive competes with her motivation to exploit her information advantage.

2.2 Special case – Risk-neutral uninformed traders

A useful special case for characterizing price sensitivities to informed demands in closed form is to assume uninformed traders' are risk neutral; i.e., $A = 0$:

Corollary 1.1:

If uninformed traders are risk-neutral, then the following $\boldsymbol{\Lambda}_i$ satisfies the equilibrium conditions:

$$\boldsymbol{\Lambda}_i = \sqrt{\tfrac{M}{M-1}}\boldsymbol{\Sigma}_x^{-1/2}\left(\boldsymbol{\Sigma}_x^{1/2}\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_x^{1/2}\right)\boldsymbol{\Sigma}_x^{-1/2}, \tag{16}$$

which implies the following expected returns and informed demands:

$$\mathrm{E}[\boldsymbol{v}-\boldsymbol{p}] = \tfrac{1}{2M}\boldsymbol{\Lambda}_i\bar{\boldsymbol{x}}, \qquad \boldsymbol{y} = \tfrac{1}{2M}\bar{\boldsymbol{x}} + \tfrac{1}{2}\boldsymbol{\Lambda}_i^{-1}\left(\bar{\boldsymbol{v}}_i - \bar{\boldsymbol{v}} + \boldsymbol{\Lambda}_i(\boldsymbol{x}-\bar{\boldsymbol{x}})\right). \tag{17}$$

The expression for $\boldsymbol{\Lambda}_i$ resembles the solution from Caballé and Krishnan (1994).[11] The first term in the informed trade $\boldsymbol{y}$ in (17) reflects the informed trader's average holdings. In the large economy limit $(N, M \to \infty)$, expected returns approach zero and the informed trader bears none of the expected supply $\bar{\boldsymbol{x}}$. When uninformed traders are risk-averse, the informed trader extracts some of the risk premium by bearing some of the expected supply even in the limit. The informed trader also absorbs some of the unexpected supply in this case.

[11] Informed traders in Caballé and Krishnan (1994) submit market orders to perfectly competitive market makers. Their expression for $\boldsymbol{\Lambda}_i$ is equivalent to expression (16) after removing the $\sqrt{\tfrac{M}{M-1}}$ term and multiplying by one half. As in Caballé and Krishnan (1994) , it is possible that there exists another, non-symmetric, matrix $\boldsymbol{\Lambda}_i$ satisfying the equilibrium conditions.

2.3 Extreme cases of market liquidity with price taking uninformed traders

In section 3, we take the large economy limit. Here we consider the effects of market liquidity given finite assets assuming that uninformed traders are price takers. The price-taking assumption removes their price impact matrix, $\boldsymbol{\Lambda}_u$, from the expressions (11) and (12) for prices and expected returns, and from the equilibrium relation that determines $\boldsymbol{\Lambda}_i$.[12] This eases the analysis allowing us to examine the role of noise trade volatility as a disguise for the informed trader's demands by comparing the extremes of uninformative prices versus certain noise trade. We cannot examine the certain noise trade case when uninformed traders consider their price impact because the market breaks down as in Kyle (1989, p. 335). When there is no uncertainty about noise trade, the price impact matrices $\boldsymbol{\Lambda}_i$ and $\boldsymbol{\Lambda}_u$ become unbounded and both informed and uninformed traders take positions approaching zero, as can be seen by the inversion of the price impact matrices in (7) and (10).

In the case of price-taking uninformed traders, we have the following corollary:

Corollary 1.2:

If uninformed traders are price-takers, then as noise trade variance becomes unbounded and price becomes uninformative $(\boldsymbol{\Sigma}_x \to \infty)$*:*

$$\boldsymbol{\Sigma}_{v|p} \to \boldsymbol{\Sigma}_v, \quad \mathrm{E}[\boldsymbol{v}-\boldsymbol{p}] \to \tfrac{1}{2M} A\boldsymbol{\Sigma}_v \bar{\boldsymbol{x}}, \quad \boldsymbol{y} \to \tfrac{1}{2}\boldsymbol{x} + \tfrac{1}{2}\left(\tfrac{1}{M} A\boldsymbol{\Sigma}_v\right)^{-1}(\bar{\boldsymbol{v}}_i - \bar{\boldsymbol{v}}). \tag{18}$$

As noise trade becomes certain $(\boldsymbol{\Sigma}_x \to \mathbf{O})$*:*

$$\boldsymbol{\Sigma}_{v|p} \to \boldsymbol{\Sigma}_v - \tfrac{1}{2}\boldsymbol{\Sigma}_{\bar{v}_i}, \quad \mathrm{E}[\boldsymbol{v}-\boldsymbol{p}] \to \tfrac{1}{2M} A(\boldsymbol{\Sigma}_v + \boldsymbol{\Sigma}_i)\bar{\boldsymbol{x}}, \quad \boldsymbol{y} \to \tfrac{1}{2}(\boldsymbol{x} - \bar{\boldsymbol{x}}) \approx \mathbf{0}. \tag{19}$$

The extreme noise-trade-variance cases provide closed-form solutions for $\boldsymbol{\Lambda}_i$. Uninformed traders learn more as noise trade becomes certain and the informed trader reveals

[12] Alternatively, we could assume that risk-aversion A increases in proportion to the number M of uninformed traders, and take limits as $M \to \infty$. This is analogous to the Kyle's (1989, section 8) analysis of a market with free entry of uninformed traders.

half of her information notwithstanding that her demands become infinitesimally small. Expected returns are higher in this case because, as the informed trades approach zero, virtually all of the payoff risk is borne by uninformed traders for which they require a premium. As in Kyle (1989), the informed trader reduces the size of her demands, but the price reaction to her trades becomes large implying that uninformed traders learn more. Comparing expected returns in (19) to (18), we see that expected returns are lower in the case where uninformed traders learn nothing. This occurs because infinite noise trade variance provides infinite disguise, which induces the informed trader to trade more aggressively thereby absorbing more risk. The greater risk absorption more than compensates for the diminished learning from prices. In section 4, we provide a setting where expected returns are strictly decreasing in noise trade variance.

3 Large economy limit

We characterize a linear equilibrium for a large economy by letting $N, M \to \infty$, with N / M approaching a finite constant which, without loss of generality, we assume to be one.[13] In doing so, we exploit an approximate factor structure to characterize the limiting behavior of the payoffs' covariance matrix. Chamberlain and Rothschild (1983) define an approximate *K*-factor structure as follows:[14]

Definition: *The sequence* $\{\boldsymbol{\Sigma}_{vN}\}_{N=1}^{\infty}$ *of* $N \times N$ *covariance matrices has an approximate factor structure if there exists a sequence of* $1 \times K$ *row vectors* $\{\boldsymbol{b}'_n\}_{n=1}^{\infty}$ *and a sequence* $\{\boldsymbol{\Sigma}_{eN}\}_{N=1}^{\infty}$ *of* $N \times N$ *positive semi-definite matrices whose eigenvalues are uniformly bounded by* $\overline{\theta} \in (0, \infty)$ *such that:*

[13] The assumption that N / M approaches a constant means that both N and M grow at the same rate; otherwise, the ratio N / M approaches either zero or infinity. Hughes, Liu, and Liu (2007), Lambert, Leuz and Verrecchia (2007), and Ou-Yang (2005) employ similar assumptions.

[14] Our discussion pertains to the existence of an approximate factor structure from the perspective of investors, rather than to the econometrician's problem of identifying a factor structure. See Gilles and LeRoy (1991) and Lewellen et al. (2010) for discussions of issues that arise in testing particular factor models.

$$\boldsymbol{\Sigma}_{vN} = \boldsymbol{B}_N \boldsymbol{B}_N' + \boldsymbol{\Sigma}_{eN}, \tag{20}$$

for all N where $\boldsymbol{b}_n'$ is the n^{th} row of the $N \times K$ matrix $\boldsymbol{B}_N$.

Chamberlain (1983) and Chamberlain and Rothschild (1983) show that an approximate *K*-factor structure is equivalent to having the first *K* eigenvalues of $\boldsymbol{\Sigma}_{vN}$ becoming unbounded as $N \to \infty$ while the remaining eigenvalues remain positive (no redundant assets) and finite (any unbounded eigenvalues are included in the *K* factors). The following remark shows that an approximate factor structure holds under the weak assumption that the payoff from a finite investment in an equally weighted portfolio has finite variance as $N \to \infty$:

Remark: *If the equally weighted portfolio has positive, but bounded, variance absent any prior information* $\left(\lim_{N\to\infty} \frac{1}{N} \operatorname{var}\left(\sum_{n=1}^{N} v_n\right) \in (0,\infty)\right)$, *then* $\boldsymbol{\Sigma}_v$ *has an approximate K-factor structure.*

Assuming that $\boldsymbol{\Sigma}_v$ has an approximate factor structure, then we can write the payoff vector $\boldsymbol{v}$ in the following form:

$$\boldsymbol{v} = \overline{\boldsymbol{v}} + \boldsymbol{B}\boldsymbol{f} + \boldsymbol{e}, \tag{21}$$

where $\boldsymbol{f}$ is *K*-dimensional standard normal random vector and $\boldsymbol{e}$ is an *N*-dimensional random vector with mean zero and covariance matrix $\boldsymbol{\Sigma}_e$. We do not restrict the informed trader's information to either systematic or idiosyncratic risks.

The following proposition characterizes expected returns in the limiting economy up to the matrix of sensitivities of prices to informed demands $\boldsymbol{\Lambda}_i$.

Proposition 2:

Assume that the informed trader's second-order condition is satisfied in the limiting economy. Then, the risk premium approaches the following as $N, M \to \infty$, and depends only on *systematic risks:*

$$\mathrm{E}[\boldsymbol{v}-\boldsymbol{p}] \to A\tfrac{1}{M}\boldsymbol{B}_p\boldsymbol{\Sigma}_{f|p}\boldsymbol{B}_p'\left(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i'\right)^{-1}\left(\boldsymbol{I}+\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1}\right)\boldsymbol{\Lambda}_i'\bar{\boldsymbol{x}}, \tag{22}$$

where $\boldsymbol{B}_p = \boldsymbol{B} - \mathrm{cov}(\boldsymbol{e},\boldsymbol{p})\,\mathrm{var}(\boldsymbol{p}\,|\,\boldsymbol{f})^{-1}\,\mathrm{cov}(\boldsymbol{p},\boldsymbol{f})$ *is the* $N\times K$ *matrix of conditional factor loadings and the* $K\times 1$ *vector of factor risk premiums is*

$$A\tfrac{1}{M}\boldsymbol{\Sigma}_{f|p}\boldsymbol{B}_p'\left(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i'\right)^{-1}\left(\boldsymbol{I}+\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1}\right)\boldsymbol{\Lambda}_i'\bar{\boldsymbol{x}}. \tag{23}$$

It is clear from (22) that information asymmetry affects expected returns only via the factor loadings $\boldsymbol{B}_p$ and factor risk premiums as depicted by (23). This implies that differences in firms' betas explain any cross-sectional variation in expected returns. Information asymmetry does not create any new factors, but instead impacts the pricing of the systematic components of the firms' fundamental payoffs.

Proposition 2 extends previous results in a perfectly competitive setting by Hughes. et al. (2007) and Lambert, et al. (2007) to an imperfectly competitive setting in which a large informed trader with a monopoly on private information seeks to take advantage of that information. Risk neutrality on the part of the informed trader along with a monopoly position implies a strong incentive for distorting trades away from a diversified portfolio. However, expected returns are driven by the risk premium required by well-diversified uninformed traders. Competition among uninformed traders eliminates their incentive to deviate from a diversified portfolio, so that expected returns reflect only non-diversifiable risk.

Risk neutrality also adds a further dimension by creating an incentive for the informed trader to extract part of the factor risk premium. Greater noise trade volatility reduces the informed trader's concern about revelation of her private information enabling greater absorption of systematic risk and lowering expected returns required by uninformed traders for bearing the remainder. This differs from the competitive setting in which aggregate demands by a continuum of risk-averse privately informed traders reduce systematic risk premiums. If the informed trader were risk-averse, the uninformed traders would continue to hold diversified portfolios so that expected returns reflect only systematic risk. The results would differ in that a single, risk-averse informed trader's risk-bearing capacity would be negligible in the large economy limit. As a result, her trades in systematic risk would be negligible, neither revealing information, nor relieving the risk burden on the uninformed traders. The negligible informed trades in systematic risk would result in expected returns that match an economy with no informed trader.

4 Additional structure on private information and noise trades

4.1 Finite economy

In this section, we impose additional structure on the informed trader's private information and noise trades that enables us to both show the existence and uniqueness of a linear equilibrium with a closed-form solution for the sensitivities of prices to informed demands, and perform additional analysis of expected returns and those demands. Specifically, we require that the informed trader's private information does not alter the covariance structure between asset payoffs, and that noise trades include trades in factor portfolios.[15] This structure makes it possible to decompose trades in the limit by risk sources (both systematic and idiosyncratic) and allows us to more fully portray the effects of the impact of private information and market liquidity as defined by the volatility of noise trades on expected return.

If there are no redundant assets, we can, without loss of generality, write the covariance matrix of $\boldsymbol{v}$ as $\boldsymbol{\Sigma}_v = \boldsymbol{T\Theta}_v\boldsymbol{T}'$ where $\boldsymbol{T}$ is an orthonormal matrix of eigenvectors ($\boldsymbol{T'T} = \boldsymbol{TT'} = \boldsymbol{I}$) and $\boldsymbol{\Theta}_v$ is a diagonal matrix containing the N strictly positive eigenvalues of $\boldsymbol{\Sigma}_v$. Denoting the

[15] These assumptions are made precise below applying principal components analysis as used in section 3 in deriving a factor structure for asset payoffs. See Van Nieuwerburgh and Veldkamp (2010) and Banerjee (2011) for examples of this approach in settings with price-taking investors.

k^{th} largest eigenvalue by θ_k, $\theta_1 > \theta_2 > \cdots > \theta_N > 0$, we can write $\boldsymbol{\Sigma}_v = \sum_{k=1}^{N} \theta_k \boldsymbol{t}_k \boldsymbol{t}_k'$, where $\boldsymbol{t}_k$ denotes the k^{th} column of $\boldsymbol{T}$.[16] A portfolio is a linear combination of the individual assets' payoffs. Traders can trade a portfolio with a payoff $f_k = \theta_k^{-1/2} \boldsymbol{t}_k' (\boldsymbol{v} - \overline{\boldsymbol{v}})$ with price $p_{fk} = \theta_k^{-1/2} \boldsymbol{t}_k' (\boldsymbol{p} - \overline{\boldsymbol{v}})$ that isolates the risk represented by the k^{th} eigenvalue, with $\text{var}(f_k) = \theta_k^{-1} \boldsymbol{t}_k' \boldsymbol{\Sigma}_v \boldsymbol{t}_k = \sum_{j=1}^{N} \theta_k^{-1} \theta_j \boldsymbol{t}_k' \boldsymbol{t}_j \boldsymbol{t}_j' \boldsymbol{t}_k = 1$ and $\text{cov}(f_k, f_j) = 0$ for $j \neq k$.[17] The N orthogonal portfolio payoffs in the vector $\boldsymbol{f} = \boldsymbol{\Theta}_v^{-1/2} \boldsymbol{T}'(\boldsymbol{v} - \overline{\boldsymbol{v}})$ are the principal components of $\boldsymbol{v}$, which we refer to as 'factors' for the sake of brevity. By construction, $\text{var}(\boldsymbol{f}) = \boldsymbol{I}$.

We make two assumptions that allow us to derive an expression for $\boldsymbol{\Lambda}_i$ in terms of trade in each factor. First, we assume that the covariance matrix of the informed trader's posterior mean $\overline{\boldsymbol{v}}_i$ has the same eigenvectors as $\boldsymbol{\Sigma}_v$, $\text{var}(\overline{\boldsymbol{v}}_i) = \boldsymbol{T}\boldsymbol{\Theta}_i\boldsymbol{T}'$, where $\boldsymbol{\Theta}_i \leq \boldsymbol{\Theta}_v$ is a diagonal matrix. In other words, the informed trader's posterior variance, $\boldsymbol{\Sigma}_i = \boldsymbol{\Sigma}_v - \boldsymbol{\Sigma}_{\overline{v}_i} = \boldsymbol{T}(\boldsymbol{\Theta}_v - \boldsymbol{\Theta}_i)\boldsymbol{T}'$. As mentioned above, this assumption implies that the informed trader's information does not alter the covariance structure between assets. It applies, for example, if the informed trader obtains a signal $\boldsymbol{v} + \boldsymbol{\eta}$ where $\boldsymbol{\eta}$ is normally distributed, independent of $\boldsymbol{v}$, with a covariance matrix having eigenvectors $\boldsymbol{T}$. Second, we assume that the noise trade covariance matrix has the eigenvectors $\boldsymbol{T}$ so that $\boldsymbol{\Sigma}_x = \boldsymbol{T}\boldsymbol{\Theta}_x\boldsymbol{T}'$, $\boldsymbol{\Theta}_x$ diagonal, consistent with noise traders transacting directly in the factor portfolios. We denote by θ_{ki} the k^{th} diagonal element of $\boldsymbol{\Theta}_i$, corresponding to the informed trader's information on the k^{th} factor, whose prior variance is θ_k. We denote by θ_{kx} the k^{th}

[16] This decomposition can be recast as a K-factor structure $\boldsymbol{B}\boldsymbol{B}' + \boldsymbol{\Sigma}_e$ by putting $\boldsymbol{B}\boldsymbol{B}' = \sum_{k=1}^{K} \theta_k \boldsymbol{t}_k \boldsymbol{t}_k'$ and $\boldsymbol{\Sigma}_e = \sum_{k=K+1}^{N} \theta_k \boldsymbol{t}_k \boldsymbol{t}_k'$ or, equivalently, $\boldsymbol{B} = \boldsymbol{T}_K \boldsymbol{\Theta}_K^{1/2}$ and $\boldsymbol{\Sigma}_e = \boldsymbol{T}_e \boldsymbol{\Theta}_e \boldsymbol{T}_e'$ where $\boldsymbol{T}_K$ ($\boldsymbol{T}_e$) contains the first K columns of $\boldsymbol{T}$ (columns $K+1, \ldots, N$ of $\boldsymbol{T}$) and $\boldsymbol{\Theta}_K$ ($\boldsymbol{\Theta}_e$) is the upper K×K quadrant of $\boldsymbol{\Theta}_v$ (lower (N-K)×(N-K) quadrant of $\boldsymbol{\Theta}_v$.

[17] The equality follows because the eigenvectors are orthonormal or, equivalently, $\boldsymbol{t}_k' \boldsymbol{t}_k = 1$ and $\boldsymbol{t}_k' \boldsymbol{t}_j = 0$ for $j \neq k$.

diagonal element of $\boldsymbol{\Theta}_x$, corresponding to the variance of noise trade in the portfolio with payoffs $f_k = \theta_k^{-1/2}\boldsymbol{t}_k'(\boldsymbol{v}-\overline{\boldsymbol{v}})$ that represents the k^{th} factor.

Under these assumptions, we obtain the following expression for expected returns:

Proposition 3:

If the covariance matrix of the informed trader's posterior mean and noise trade satisfy $\boldsymbol{\Sigma}_{\overline{v}_i} = \boldsymbol{T\Theta}_i\boldsymbol{T}'$ and $\boldsymbol{\Sigma}_x = \boldsymbol{T\Theta}_x\boldsymbol{T}'$, $\boldsymbol{\Theta}_i$ and $\boldsymbol{\Theta}_x$ diagonal, then the expected return on the k^{th} factor and the vector of expected returns are:

$$\mathrm{E}[f_k - p_{fk}] = \underbrace{\left(\tfrac{1}{2M-1}\lambda_k + A\tfrac{1}{M}\theta_{k|p}\right)}_{\substack{\text{Effect of risk aversion and}\\ \text{imperfect competition among}\\ \text{uninformed traders}}}\ \underbrace{\tfrac{1}{2}\left(1+\tfrac{\theta_{ki}}{\lambda_k^2\theta_{kx}}\right)}_{\substack{\text{Effect of informed}\\ \text{trader's absorption}\\ \text{of risk associated}\\ \text{expected noise trades}}}\ \overline{x}_{fk} = \tfrac{1}{2}\left(1-\tfrac{\theta_{ki}}{\lambda_k^2\theta_{kx}}\right)\lambda_k\overline{x}_{fk},$$

$$\mathrm{E}[\boldsymbol{v}-\boldsymbol{p}] = \boldsymbol{T\Theta}_v^{-1/2}\,\mathrm{E}[\boldsymbol{f}-\boldsymbol{p}_f], \tag{24}$$

where $\theta_{k|p} = \theta_k - \frac{\theta_{ki}^2}{\theta_{ki}+\lambda_k^2\theta_{kx}}$ is the uninformed traders' posterior variance for the k^{th} factor, and $\lambda_k > 0$ is the k^{th} element of the diagonal matrix $\boldsymbol{\Theta}_\lambda = \boldsymbol{T}'\boldsymbol{\Lambda}_i\boldsymbol{T}$ and $\overline{x}_{fk} = \theta_k^{-1/2}\boldsymbol{t}_k'\overline{\boldsymbol{x}}$. The informed demand in the k^{th} factor is:

$$y_{fk} = \tfrac{1}{2}\left(\left(1-\tfrac{\theta_{ki}}{\lambda_k^2\theta_{kx}}\right)\overline{x}_{fk} + \tfrac{1}{\lambda_k}\overline{f}_{ki} + x_{fk} - \overline{x}_{fk}\right), \tag{25}$$

where $\overline{f}_{ki} = \theta_k^{-1/2}\boldsymbol{t}_k'(\overline{\boldsymbol{v}}_i - \overline{\boldsymbol{v}})$ and $x_{fk} = \theta_k^{-1/2}\boldsymbol{t}_k'\boldsymbol{x}$.

The following corollary gives the effect of noise trade variance on expected returns and sensitivity of price to informed demands λ_k. This effect plays a role in deriving the large-economy expected returns.

Corollary 3.1:

The price impact $\lambda_k > \sqrt{\frac{\theta_{ki}}{\theta_{kx}}}$. *The uninformed traders' posterior uncertainty* $\theta_{k|p}$ *is increasing in* θ_{kx}. *Expected returns* $\mathrm{E}[f_k - p_{fk}]$ *and the price impact* λ_k *are both decreasing in the variance* θ_{kx} *of noise trades. The expected informed demand* $\mathrm{E}[y_{fk}]$ *is increasing in* θ_{kx}.

Taking limits, as $\theta_{kx} \to \infty$ $(\theta_{kx} \to 0)$, $\lambda_k \to A\frac{2M-1}{2(M-1)}\frac{1}{M}\theta_k$ (λ_k *becomes unbounded at rate* $\theta_{kx}^{-1/2}$), $\mathrm{E}[f_k - p_{fk}] \to \frac{1}{2}\frac{2M-1}{2(M-1)}A\frac{1}{M}\theta_k\overline{x}_k$ ($\mathrm{E}[f_k - p_{fk}]$ *becomes unbounded*), *and* $y_{fk} \to \frac{1}{2}\left(x_{fk} + \frac{2(M-1)}{2M-1}\frac{1}{A\frac{1}{M}\theta_k}\overline{f}_{ki}\right)$ $\left(y_{fk} \to \frac{1}{2}\left(x_{fk} - \frac{M-1}{M}\overline{x}_{fk}\right)\right)$.

Increases in noise trade variance both reduce the information revealed through price ($\theta_{k|p}$ increases in θ_{kx}) and increase the amount of risk associated with noise trades absorbed by the informed trader. Corollary 3.1 implies that the increase in the risk-neutral informed trader's risk-absorption dominates. Expected returns decline as the variance of noise trade increases, even though uninformed traders face greater uncertainty.

In this setting, the effect of distorted risk-sharing is so strong that *any* private information increases expected returns, as we state in the following corollary:

Corollary 3.2

Any private information $(\theta_{ki} > 0)$ *yields higher expected returns than no private information. Formally,* $\mathrm{E}[f_k - p_{fk}; \theta_{ki} \neq 0] > \mathrm{E}[f_k - p_{fk}; \theta_{ki} = 0]$ *for any* $\theta_{ki} \in (0, \theta_k]$.

The relation between private information θ_{ki} and expected returns is non-monotonic; however, the above corollary shows that expected returns are lower if there was no private information at all. This stands in contrast to settings where informed traders are risk averse and perfectly competitive, and any information that increases the average precision of traders' beliefs yields a reduction in expected returns (Lambert et al. 2012).

4.2 Large economy limit

We take the large economy limit for the above setting where traders trade portfolios that isolate risk sources. Expression (24) gives the expected returns of $\mathrm{E}[\boldsymbol{v}-\boldsymbol{p}]=\boldsymbol{T\Theta}_v^{-1/2}\,\mathrm{E}[\boldsymbol{f}-\boldsymbol{p}_f]$ in the finite economy. In this setting, we essentially have factor pricing, where $\boldsymbol{T\Theta}_v^{-1/2}$ plays the role of the factor loadings $\boldsymbol{B}$. The following proposition shows that, when $\boldsymbol{\Sigma}_v$ has an approximate K-factor structure, only the systematic factors are priced in the large economy $(N,M\to\infty)$.

Proposition 4:

If the covariance matrix $\boldsymbol{\Sigma}_v=\boldsymbol{T\Theta}_v\boldsymbol{T}'$ *has an approximate K-factor structure, and the covariance matrix of the informed trader's posterior mean and of noise trade satisfy* $\boldsymbol{\Sigma}_{\overline{v}_i}=\boldsymbol{T\Theta}_i\boldsymbol{T}'$ *and* $\boldsymbol{\Sigma}_x=\boldsymbol{T\Theta}_x\boldsymbol{T}'$, $\boldsymbol{\Theta}_i$ *and* $\boldsymbol{\Theta}_x$ *diagonal, then, as the economy expands* $(N,M\to\infty)$*, the expected return and informed trade* $y_k=\theta_k^{-1/2}\boldsymbol{t}_k'\boldsymbol{y}$ *on the k^th^* <u>*systematic factor*</u> $(k\le K)$ *are as follows assuming that the variance* θ_{kx} *of noise trades for the k^th^ factor grows with the economy (at order N):*

$$\mathrm{E}[f_k-p_{fk}]\to\tfrac{1}{2}\left(1+\tfrac{\theta_{ki}}{\lambda_k^2\theta_{kx}}\right)A\tfrac{1}{M}\theta_{k|p}\overline{x}_{fk},\qquad y_{fk}\to\tfrac{1}{2}\left(\left(1-\tfrac{\theta_{ki}}{\lambda_k^2\theta_{kx}}\right)\overline{x}_{fk}+\tfrac{1}{\lambda_k}\overline{f}_{ki}+x_{fk}-\overline{x}_{fk}\right),\quad (26)$$

Relaxing the assumption on the variance of noise trades such that it grows slower than the economy, the expected return and informed demand on the k^th^ systematic factor are:

$$\mathrm{E}[f_k-p_{fk}]\to A\tfrac{1}{M}\theta_{k|p}\overline{x}_{fk}=A\tfrac{1}{M}\left(\theta_k-\tfrac{1}{2}\theta_{ki}\right)\overline{x}_{fk},\qquad y_{fk}\to\tfrac{1}{2}(x_{fk}-\overline{x}_{fk}).\qquad (27)$$

The expected returns on <u>*idiosyncratic factors*</u> $(k>K)$ *all approach zero and the corresponding informed demand approaches* $y_{fk}\to\frac{1}{2\lambda_k}\overline{f}_{ki}+\frac{1}{2}(x_{fk}-\overline{x}_{fk})$, *where* $\lambda_k\to\sqrt{\frac{\theta_{ki}}{\theta_{kx}}}$

$(\lambda_k \to 0)$ *and the information-based portion of informed demand approaches* $\frac{1}{2}\sqrt{\frac{\theta_{kx}}{\theta_{ki}}}\,\bar{f}_{ki}$ (*infinity*) *if the corresponding variance* θ_{kx} *of noise trades remains bounded* (*becomes unbounded*).

Analogous to Proposition 2, Proposition 4 implies that only systematic risks are priced in a large economy. Furthermore, it highlights the key role played by noise traders. In particular, if noise trades in systematic factor portfolios do not grow with the economy, the informed trader has no disguise for her demands. Similar to Corollary 3.1, this drives down the magnitude of informed demands. The proof of Corollary 3.1 shows that the noise in price, $\lambda_k^2\theta_{kx}$, is increasing in θ_{kx} so that noise trades reduce the information available to uninformed traders; however, the effect of reduced risk-absorption by the informed trader dominates. This implies that the sharing of systematic risks depends heavily on the extent to which noise traders transact in diversified portfolios. For example, liquidity (or irrationally) motivated trading activity in index funds provides disguise that can facilitate informed traders' bearing of systematic risks.

The following corollary gives expected returns in terms of the original securities:

Corollary 4.1:

If the $\boldsymbol{\Sigma}_v = \boldsymbol{T\Theta}_v\boldsymbol{T}'$ *has an approximate K-factor structure, and the covariance matrix of the informed trader's posterior mean and of noise trade satisfy* $\boldsymbol{\Sigma}_{\bar{v}_i} = \boldsymbol{T\Theta}_i\boldsymbol{T}'$ *and* $\boldsymbol{\Sigma}_x = \boldsymbol{T\Theta}_x\boldsymbol{T}'$, $\boldsymbol{\Theta}_i$ *and* $\boldsymbol{\Theta}_x$ *diagonal, then, as the economy expands* $(N, M \to \infty)$, *the expected return on the nth security is:*

$$\mathrm{E}[\boldsymbol{v} - \boldsymbol{p}] = \boldsymbol{T}_K\boldsymbol{\Theta}_K^{-1/2}\mathrm{E}[\boldsymbol{f}_K - \boldsymbol{p}_{fK}], \tag{28}$$

where $\boldsymbol{T}_K$ *includes the first K columns of T,* $\boldsymbol{\Theta}_K^{-1/2}$ *is the upper* $K \times K$ *block of* $\boldsymbol{\Theta}_v^{-1/2}$, *and* $\mathrm{E}[\boldsymbol{f}_K - \boldsymbol{p}_{fK}]$ *contains the first K elements of* $\mathrm{E}[\boldsymbol{f} - \boldsymbol{p}_f]$ *– the premiums for systematic risks.*

Similar to the large-economy analysis in section 3, we can express the payoff vector $\boldsymbol{v}$ as

$\boldsymbol{v} = \overline{\boldsymbol{v}} + \boldsymbol{T}\boldsymbol{\Theta}_v^{1/2}\boldsymbol{f} = \overline{\boldsymbol{v}} + \boldsymbol{B}\boldsymbol{f}_K + \boldsymbol{e}$ where $\boldsymbol{f}_K$ denotes the factors that grow with the economy, $\boldsymbol{B} = \boldsymbol{T}_K\boldsymbol{\Theta}_K^{1/2}$ is the $N \times K$ matrix of loadings on systematic risks $(\boldsymbol{B}\boldsymbol{f}_K = \sum_{k=1}^{K}\theta_k^{1/2}\boldsymbol{t}_k f_k)$, and $\boldsymbol{e} = \boldsymbol{T}_e\boldsymbol{\Theta}_e^{1/2}\boldsymbol{f}_e$ is the vector reflecting the payoff's sensitivity to the $N-K$ idiosyncratic risks $(\boldsymbol{e} = \sum_{k=K+1}^{N}\theta_k^{1/2}\boldsymbol{t}_k f_k)$. In the special setting of Proposition 4, the securities maintain their original factor loadings, which is due to the isolation of each risk source.[18]

5 Conclusion

Our analysis makes a strong case for concluding that, in a large economy where idiosyncratic risks are fully diversifiable, imperfect competition impacts expected returns solely through its effects on systematic risk premiums and assets' exposures to systematic risks (factor loadings). As in perfectly competitive settings, factor loadings fully reflect any cross-sectional differences in firms' expected returns.

Assuming CARA utility, normal distributions, and a finite economy, we derive characterizations of a linear equilibrium in which a risk-neutral informed trader and risk-averse uninformed traders are strategic in setting their demands. The informed trader's equilibrium demands reflect a tension between her exploitation of an information advantage and incentive to capture a portion of the risk premium by absorbing noise trades. The incentive to mask private information causes the informed trader to reduce her absorption of risk associated with the noisy supply of assets, leaving more risk to be absorbed by uninformed traders for which they require greater expected returns. A similar qualitative result can be obtained in finite economies with a risk-averse informed trader so long as she is sufficiently risk tolerant.

Mild restrictions on the covariance matrix of assets payoffs imply that the asset payoffs

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[18] The matrix $\boldsymbol{T}_K\boldsymbol{\Theta}_K^{-1/2}$ is analogous to the loadings $\boldsymbol{B}$ on systematic risks in our main setting. The extra term subtracted from $\boldsymbol{B}$ in $\boldsymbol{B}_p$ subtracted in Proposition 3 equals zero in the setting of Proposition 4.

approximately have a factor structure in the large economy limit. In the large economy limit, we show that only systematic risks are priced. Although our characterization of equilibrium expected returns and informed trades are up to a matrix of price sensitivities to the informed trader's demands, we derive a closed-form solution for a setting in which trade occurs in portfolios that isolate distinct sources of risk. This setting allows us to provide additional analysis on how noise trades and private information impact expected returns. We show that greater liquidity, as measured by the variance of noise trades, leads to lower expected returns even though it reduces price informativeness. The lower returns stem from the noise trades allowing the informed trader to absorb more risk without revealing more private information.[19]

Our results suggest a new avenue for empirical study that seeks to associate cost of capital with asymmetric information. To the best of our knowledge, empirical work has yet to explore the inverse relation between expected return and market liquidity as measured by the volatility of noise trades, in the presence of asymmetric information among market participants. *Ceteris paribus* variations in liquidity across markets or across time given shocks that alter liquidity raise the prospect of detecting such a relation.

[19] This relation disappears in the large economy limit when the informed trader is risk averse and, unlike the case in which there is a continuum of competitive informed traders, has limited risk bearing capacity.

References

Akins, B., J. Ng, and R. Verdi. 2012. Investor competition over information and the pricing of information asymmetry. *The Accounting Review* 87(1): 35-58.

Armstrong, C., J. Core, D. Taylor, and R. Verrecchia. 2011. When does information asymmetry affect the cost of capital. *Journal of Accounting Research* 49(1): 1-40.

Back, K., H. Cao, and G. Willard. 2000. Imperfect competition among informed traders. *Journal of Finance* 55(5): 2117-2155.

Banerjee, S. 2011. Learning from prices and the dispersion in beliefs. *Review of Financial Studies* 24(9): 3025-3068.

Caballé, J., and M. Krishnan. 1994. Imperfect competition in a multi-security market with risk neutrality. *Econometrica* 62(3), 695-704.

Caskey, J., and J. Hughes. 2012. Assessing the impact of alternative fair value measures on the efficiency of project selection and continuation. *The Accounting Review* 87(2): 483-512.

Chamberlain, G. 1983. Funds, factors, and diversification in arbitrage pricing models. *Econometrica* 51(5): 1305-1323.

Chamberlain, G., and M. Rothschild. 1983. Arbitrage, factor structure, and mean-variance analysis on large asset markets. *Econometrica* 51(5): 1281-1304.

Christensen, P., G. Feltham, and M. Wu. 2002. 'Cost of capital' in residual income for performance evaluation. *The Accounting Review* 77(1): 1-24.

Diamond, D., and R. Verrecchia. 1991. Disclosure, liquidity, and the cost of capital. *Journal of Finance* 46(4): 1325-1359.

Faccio, M., M. Marchica, and R. Mura. 2011. Large shareholder diversification and corporate risk-taking. *Review of Financial Studies* 24(11): 3601-3641.

Gao, P., and R. Verrecchia. 2012. Economic consequences of idiosyncratic information in diversified markets. Working paper, University of Chicago and University of Pennsylvania.

Gilles, C., and S. LeRoy. 1991. On the arbitrage pricing theory. *Econometric Theory* 1(3): 213-229.

Hughes, J., J. Liu, and J. Liu. 2007. Information asymmetry, diversification, and the cost of capital. *The Accounting Review* 82(3): 705-729.

Kyle, A. 1985. Continuous auctions and insider trading. *Econometrica* 53(6): 1315-1335.

Kyle, A. 1989. Informed speculation with imperfect competition. *Review of Economic Studies* 56(3): 317-355.

Lambert, R., C. Leuz, and R. Verrecchia. 2007. Accounting information, disclosure, and the cost of capital. *Journal of Accounting Research* 45(2) 385-420.

Lambert, R., C. Leuz, and R. Verrecchia. 2012. Information asymmetry, information precision, and the cost of capital. *Review of Finance* 16(1): 1-29.

Lewellen, J., S. Nagel, and J. Shanken. 2010. A skeptical appraisal of asset pricing tests. *Journal of Financial Economics* 96(2): 175-194.

Ou-Yang, H. 2005. Asset pricing and moral hazard. *Review of Financial Studies* 18(4): 1253-1303.

Subrahmanyam, A. 1991. A theory of trading in stock index futures. *Review of Financial Studies* 4(1): 17-51.

Van Nieuwerburgh, S., and L. Veldkamp. 2010. Information acquisition and under-diversification. *Review of Economic Studies* 77(2): 779-805.

Appendix

Proof of Proposition 1

The conjectured coefficients we need to specify are $\boldsymbol{q}_0, \boldsymbol{Q}_v$, and $\boldsymbol{Q}_p$, from the uninformed traders' conjecture (2) of informed trade; $\boldsymbol{c}_0$ and $\boldsymbol{C}_p$ from the uninformed traders' conjecture (3) of other uninformed traders' positions; $\boldsymbol{\mu}_i$ ($\boldsymbol{\mu}_u$) and $\boldsymbol{\Lambda}_i$ ($\boldsymbol{\Lambda}_u$) from the informed (uninformed) trader's residual supply curve. We express the equilibrium up to the solution for the informed trader's liquidity matrix $\boldsymbol{\Lambda}_i$. Matching coefficients between the informed trader's conjectured strategy (2) and chosen strategy (7) yields $\boldsymbol{Q}_v = \boldsymbol{Q}_p = (\boldsymbol{\Lambda}_i')^{-1}$ and $\boldsymbol{q}_0 = (\boldsymbol{\Lambda}_i')^{-1}\overline{\boldsymbol{v}}$. Substituting the conjectured uninformed strategy (2) into the market clearing condition (5) and rearranging yields $\boldsymbol{p} = \boldsymbol{\mu}_i + \boldsymbol{\Lambda}_i \boldsymbol{y}$, where $\boldsymbol{\mu}_i = \boldsymbol{\Lambda}_i(M\boldsymbol{c}_0 - \boldsymbol{x})$ and $\boldsymbol{\Lambda}_i = \frac{1}{M}\boldsymbol{C}_p^{-1}$.

When solving the coefficients for the uninformed, we first impose homogenous strategies in the expression for the signal $\boldsymbol{s}$ in (8) by setting $\boldsymbol{d}_m = \boldsymbol{c}_0 - \boldsymbol{C}_p\boldsymbol{p}$, which gives the following after substituting $\boldsymbol{Q}_v = \boldsymbol{Q}_p, \boldsymbol{q}_0 = \boldsymbol{Q}_v\overline{\boldsymbol{v}}$ and rearranging:

$$\boldsymbol{s} = (\boldsymbol{I} + M\boldsymbol{Q}_v^{-1}\boldsymbol{C}_p)\boldsymbol{p} - \overline{\boldsymbol{v}} + \boldsymbol{Q}_v^{-1}(\overline{\boldsymbol{x}} - M\boldsymbol{c}_0). \tag{A1}$$

Substituting from (A1) into the uninformed demand (10) and rearranging gives:

$$\begin{aligned}\boldsymbol{d}_m &= (\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})^{-1}(\mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}] - \boldsymbol{p}) = (\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})^{-1}\left(\overline{\boldsymbol{v}} + \boldsymbol{\Sigma}_{\overline{v}_i}(\boldsymbol{\Sigma}_{\overline{v}_i} + \boldsymbol{\Sigma}_s)^{-1}\boldsymbol{s} - \boldsymbol{p}\right)\\ &= \underbrace{(\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})^{-1}\left(\left(\boldsymbol{I} - \boldsymbol{\Sigma}_{\overline{v}_i}(\boldsymbol{\Sigma}_{\overline{v}_i} + \boldsymbol{\Sigma}_s)^{-1}\right)\overline{\boldsymbol{v}} + \boldsymbol{\Sigma}_{\overline{v}_i}(\boldsymbol{\Sigma}_{\overline{v}_i} + \boldsymbol{\Sigma}_s)^{-1}\boldsymbol{Q}_v^{-1}(\overline{\boldsymbol{x}} - M\boldsymbol{c}_0)\right)}_{\boldsymbol{c}_0}\\ &\qquad - \underbrace{(\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})^{-1}\left(\boldsymbol{I} - \boldsymbol{\Sigma}_{\overline{v}_i}(\boldsymbol{\Sigma}_{\overline{v}_i} + \boldsymbol{\Sigma}_s)^{-1}(\boldsymbol{I} + M\boldsymbol{Q}_v^{-1}\boldsymbol{C}_p)\right)}_{\boldsymbol{C}_p}\boldsymbol{p}.\end{aligned} \tag{A2}$$

Substituting $\boldsymbol{C}_p = \frac{1}{M}\boldsymbol{\Lambda}_i^{-1}$ and $\boldsymbol{Q}_v^{-1} = \boldsymbol{\Lambda}_i'$ into the expression for $\boldsymbol{C}_p$ in (A2) gives:

$$\tfrac{1}{M}\boldsymbol{\Lambda}_i^{-1} = (\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})^{-1}\left(\boldsymbol{I} - \boldsymbol{\Sigma}_{\overline{v}_i}(\boldsymbol{\Sigma}_{\overline{v}_i} + \boldsymbol{\Sigma}_s)^{-1}(\boldsymbol{I} + \boldsymbol{\Lambda}_i'\boldsymbol{\Lambda}_i^{-1})\right). \tag{A3}$$

Substituting $\boldsymbol{\Sigma}_{v|p} = \boldsymbol{\Sigma}_v - \boldsymbol{\Sigma}_{\overline{v}_i}(\boldsymbol{\Sigma}_{\overline{v}_i} + \boldsymbol{\Lambda}_i'\boldsymbol{\Sigma}_x\boldsymbol{\Lambda}_i)^{-1}\boldsymbol{\Sigma}_{\overline{v}_i}$ into (A3) and rearranging gives the equilibrium condition that determines $\boldsymbol{\Lambda}_i$:

$$\mathbf{O} = \boldsymbol{\Lambda}_i'\boldsymbol{\Sigma}_x\boldsymbol{\Lambda}_i\boldsymbol{\Sigma}_{\bar{v}_i}^{-1}\left(\boldsymbol{\Lambda}_i - \tfrac{1}{M}\boldsymbol{\Lambda}_u'\right) - \tfrac{1}{M}A\boldsymbol{\Lambda}_i'\boldsymbol{\Sigma}_x\boldsymbol{\Lambda}_i\boldsymbol{\Sigma}_{\bar{v}_i}^{-1}\boldsymbol{\Sigma}_v - \left(\boldsymbol{\Lambda}_i' + \tfrac{1}{M}\boldsymbol{\Lambda}_u'\right) - \tfrac{1}{M}A(\boldsymbol{\Sigma}_v - \boldsymbol{\Sigma}_{\bar{v}_i}). \tag{A4}$$

Given $\boldsymbol{\Lambda}_i$, it remains to determine $\boldsymbol{c}_0$, which can be determined from (A2):

$$\begin{aligned} \boldsymbol{c}_0 &= (\boldsymbol{\Lambda}_u' + A\boldsymbol{\Sigma}_{v|p})^{-1}\left(\left(\boldsymbol{I} - \boldsymbol{\Sigma}_{\bar{v}_i}(\boldsymbol{\Sigma}_{\bar{v}_i} + \boldsymbol{\Sigma}_s)^{-1}\right)\bar{\boldsymbol{v}} + \boldsymbol{\Sigma}_{\bar{v}_i}(\boldsymbol{\Sigma}_{\bar{v}_i} + \boldsymbol{\Sigma}_s)^{-1}\boldsymbol{Q}_v^{-1}(\bar{\boldsymbol{x}} - M\boldsymbol{c}_0)\right) \\ \Rightarrow \boldsymbol{c}_0 &= \tfrac{1}{M}\boldsymbol{\Lambda}_i^{-1}(\bar{\boldsymbol{v}} + \boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1}\boldsymbol{\Lambda}_i'\bar{\boldsymbol{x}}), \end{aligned} \tag{A5}$$

where the second line follows from rearranging the first line, including a substitution from (A3).

The equilibrium coefficients, given $\boldsymbol{\Lambda}_i$, are:

$$\begin{gathered} \boldsymbol{c}_0 = \tfrac{1}{M}\boldsymbol{\Lambda}_i^{-1}(\bar{\boldsymbol{v}} + \boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1}\boldsymbol{\Lambda}_i'\bar{\boldsymbol{x}}), \quad \boldsymbol{C}_p = \tfrac{1}{M}\boldsymbol{\Lambda}_i^{-1}, \\ \boldsymbol{\mu}_u = \boldsymbol{\Lambda}_u\left((\boldsymbol{\Lambda}_i')^{-1}\bar{\boldsymbol{v}}_i + (M-1)\boldsymbol{c}_0 - \boldsymbol{x}\right), \quad \boldsymbol{\Lambda}_u = \boldsymbol{\Lambda}_i'\left(\boldsymbol{\Lambda}_i + \tfrac{M-1}{M}\boldsymbol{\Lambda}_i'\right)^{-1}\boldsymbol{\Lambda}_i, \\ \boldsymbol{q}_0 = (\boldsymbol{\Lambda}_i')^{-1}\bar{\boldsymbol{v}}, \quad \boldsymbol{Q}_v = \boldsymbol{Q}_p = (\boldsymbol{\Lambda}_i')^{-1}, \quad \boldsymbol{\mu}_i = \boldsymbol{\Lambda}_i(M\boldsymbol{c}_0 - \boldsymbol{x}). \end{gathered} \tag{A6}$$

Substituting $\boldsymbol{\mu}_i$ and (A4) into the first expression in (7) yields (13). The uninformed trader's first-order condition implies the first line of (11), while market clearing and (13) imply (14). Substituting for $\mathrm{E}[\boldsymbol{v} \mid \boldsymbol{p}] = \mathrm{E}[\boldsymbol{v} \mid \boldsymbol{s}]$ and $\boldsymbol{d}_m$ yield the second line of (11), after a substitution from (A4). ■

Proof of Corollary 1.1

If $\boldsymbol{\Lambda}_i$ is symmetric, then $\boldsymbol{\Lambda}_u = \frac{M}{2M-1}\boldsymbol{\Lambda}_i$. Setting $A = 0$ in (A4) and rearranging yields:

$$\boldsymbol{\Sigma}_x^{1/2}\boldsymbol{\Lambda}_i\boldsymbol{\Sigma}_x^{1/2}\boldsymbol{\Sigma}_x^{1/2}\boldsymbol{\Lambda}_i\boldsymbol{\Sigma}_x^{1/2} = \tfrac{M}{M-1}\boldsymbol{\Sigma}_x^{1/2}\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_x^{1/2}, \tag{A7}$$

which implies (16). Expression (17) follows from substituting $A = 0$ and (16) into (12) and (13).■

Proof of Corollary 1.2

We first prove a preliminary result that the informed trader never reveals all of her information $(\boldsymbol{\Sigma}_s > \mathbf{O})$ and that $\boldsymbol{\Sigma}_s \to \infty$ as $\boldsymbol{\Sigma}_x \to \infty$. The equilibrium equation (A4) that defines $\boldsymbol{\Lambda}_i$ can be written as:

$$\Lambda_i \left(\Lambda_i + \tfrac{M}{M-1}\Lambda_i'\right)^{-1} = \tfrac{1}{M} A\left(\Sigma_v - \Sigma_{\bar{v}_i}(\Sigma_{\bar{v}_i} + \Sigma_s)^{-1}\Sigma_{\bar{v}_i}\right)\left(\Lambda_i' + \Lambda_i\right)^{-1} + \Sigma_{\bar{v}_i}(\Sigma_{\bar{v}_i} + \Sigma_s)^{-1}. \quad \text{(A8)}$$

If $\Sigma_s \to \infty$, then (A8) can be written as $\Lambda_i \left(\Lambda_i + \tfrac{M}{M-1}\Lambda_i'\right)^{-1} = \tfrac{1}{M} A\Sigma_v \left(\Lambda_i + \Lambda_i'\right)^{-1}$, which is solved by $\Lambda_i = \tfrac{1}{M}\tfrac{2M-1}{2(M-1)} A\Sigma_v$.[20] In the price-taking uninformed case, we can write this as $\Lambda_i = \tfrac{1}{M} A\Sigma_v$. If $\Sigma_s \to \mathbf{O}$, then (A8) can be written as $\Lambda_i \left(\Lambda_i + \tfrac{M}{M-1}\Lambda_i'\right)^{-1}\left(\Lambda_i' + \Lambda_i\right) = -\tfrac{1}{M} A\left(\Sigma_v - \Sigma_{\bar{v}_i}\right)$, which has a negative definite right-hand-side. In the price-taking uninformed case, or for M sufficiently large, the left-hand-side approaches Λ_i, which must be positive definite in order to satisfy the informed trader's second-order condition. Thus, Λ_i must become unbounded at order $\Sigma_x^{-1/2}$ as $\Sigma_x \to \mathbf{O}$ so that $\Sigma_s = \Lambda_i'\Sigma_x\Lambda_i$ does not approach zero.

If the uninformed traders are price-takers, we can reflect this by dropping Λ_u from the expression (A4) that defines Λ_i and from the expression (12) for expected returns. If $\Sigma_x \to \infty$, then $\Lambda_i \to \tfrac{1}{M} A\Sigma_v$ because any bounded, nonzero Λ_i yields $\Sigma_s \to \infty$. This gives (18). If $\Sigma_x \to \mathbf{O}$, Λ_i becomes unbounded. Solving (A4) for $\Sigma_s = \Lambda_i'\Sigma_x\Lambda_i$ and taking limits as $\Lambda_i \to \infty$ for the Λ_i terms other than Σ_s yields $\Sigma_s \to \Sigma_{\bar{v}_i}$ as $\Sigma_x \to \mathbf{O}$. This gives (19), where $\boldsymbol{y} \approx \mathbf{0}$ because the variance approaching zero implies that $\boldsymbol{x} \approx \bar{\boldsymbol{x}}$, in terms of the mean squared difference approaching zero. ■

Proof of Remark

Denote the payoff from the equally-weighted market portfolio by r_{ew}. Covariance matrices are real and symmetric so that we can, without loss of generality, diagonalize Σ_{vN} as $\Sigma_{vN} = \boldsymbol{T}_N\boldsymbol{\Theta}_N\boldsymbol{T}_N' = \sum_{k=1}^{N}\theta_k \boldsymbol{t}_k\boldsymbol{t}_k'$, where $\boldsymbol{\Theta}_N$ is a diagonal matrix of eigenvalues, $\boldsymbol{T}_N$ is an orthonormal matrix of eigenvectors ($\boldsymbol{T}_N\boldsymbol{T}_N' = \boldsymbol{T}_N'\boldsymbol{T}_N = \boldsymbol{I}$), and $\boldsymbol{t}_k$ is the k^{th} column of $\boldsymbol{T}_N$. A \$1

[20] We cannot rule out additional solutions with a non-symmetric Λ_i.

investment in the equal-weighted portfolio can be represented by a vector of weights $\frac{1}{N}\mathbf{1}$ where $\mathbf{1}$ is a vector of ones. This gives:

$$\mathrm{var}(r_{ew}) = \lim_{N\to\infty} \mathrm{var}\left(\tfrac{1}{N}\mathbf{1}'\boldsymbol{v}_N\right) = \lim_{N\to\infty} \tfrac{1}{N^2}\mathbf{1}'\boldsymbol{\Sigma}_{vN}\mathbf{1} = \lim_{N\to\infty} \tfrac{1}{N^2}\sum_{k=1}^{N}\theta_k(\mathbf{1}'\boldsymbol{t}_k)^2. \tag{A9}$$

The Cauchy-Schwarz inequality implies that $(\mathbf{1}'\boldsymbol{t}_k)^2 \le (\mathbf{1}'\mathbf{1})(\boldsymbol{t}_k'\boldsymbol{t}_k) = N$, where $\boldsymbol{t}_k'\boldsymbol{t}_k = 1$ follows from the eigenvectors being orthonormal. Thus, each summand $\frac{1}{N^2}\theta_k(\mathbf{1}'\boldsymbol{t}_k)^2 \le \frac{1}{N}\theta_k$, which approaches zero for any bounded eigenvalue $\theta_k = o(N)$.[21] The elements of the eigenvectors associated with unbounded eigenvalues, $\theta_k \sim N$, have elements of order $N^{-1/2}$, which follows because $\boldsymbol{t}_k'\boldsymbol{t}_k = 1$ and the eigenvectors have nonzero elements for a nontrivial fraction of the assets.[22] Thus, the summand $\frac{1}{N^2}\theta_k(\mathbf{1}'\boldsymbol{t}_k)^2 \sim 1$ for any eigenvalue $\theta_k \sim N$. If there are K such unbounded eigenvalues, then the sum is of order K, $\mathrm{var}(r_{ew}) \sim K$. If there are no unbounded eigenvalues, then $\mathrm{var}(r_{ew}) \to 0$ while if there are infinitely many, then $\mathrm{var}(r_{ew}) \to \infty$. Thus, the positive bounded variance of a finite investment in the equal-weighted portfolio is equivalent to having K unbounded eigenvalues, which Chamberlain and Rothschild (1983) show is equivalent to having an approximate K-factor structure. ■

Proof of Proposition 2

The proof proceeds in steps. The expected returns can be written as:

$$A\underbrace{\tfrac{1}{M}\boldsymbol{\Sigma}_{v|p}}_{\text{Step 2}}\underbrace{(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I} + \boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Lambda}_i'}_{\text{Step 3}}\bar{\boldsymbol{x}} + \underbrace{\tfrac{1}{M}\boldsymbol{\Lambda}_u'}_{\text{Step 1}}\underbrace{(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I} + \boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Lambda}_i'}_{\text{Step 3}}\bar{\boldsymbol{x}}. \tag{A10}$$

Step 1 shows that the term $\frac{1}{M}\boldsymbol{\Lambda}_u'$ that reflects imperfect competition among uninformed investors vanishes in the limit. Step 2 gives the limiting expressions for $\frac{1}{M}\boldsymbol{\Sigma}_{v|p}$. Step 3 shows that the

[21] We use the notation $x_N = O(y_N)$ to denote that x_N/y_N is bounded, $x_N = o(y_N)$ denotes that $x_N / y_N \to 0$, and $x_N \sim y_N$ denote that both x_N/y_N and y_N/x_N are bounded and thus increase at roughly the same rate as $N \to \infty$.

[22] This can also be seen by noting that the variance of asset n is $\sum_{k=1}^{N}\theta_k t_{nk}^2$, which is unbounded for any individual asset that bears a fraction greater than order $N^{-1/2}$ of the risk of associated with an eigenvector of order N.

matrix that multiplies $\bar{\boldsymbol{x}}$ to get the average per capita uninformed demand $M\,\mathrm{E}[\boldsymbol{d}_m]$ is bounded, which implies that $M\,\mathrm{E}[\boldsymbol{d}_m]$ has bounded elements corresponding to the bounded supply of each share.

Step 1: $\frac{1}{M}\boldsymbol{\Lambda}_u$ *vanishes as* $N, M \to \infty$

Expression (A6) for $\boldsymbol{\Lambda}_u$, can be written as:

$$\tfrac{1}{M}\boldsymbol{\Lambda}_u = \boldsymbol{\Lambda}_i' - \tfrac{M-1}{M}\boldsymbol{\Lambda}_i'\left(\boldsymbol{\Lambda}_i + \tfrac{M-1}{M}\boldsymbol{\Lambda}_i'\right)^{-1}(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i'), \tag{A11}$$

where the right-hand-side approaches $\mathbf{O}$ as $N, M \to \infty$.

Step 2: $\frac{1}{M}\operatorname{var}(\boldsymbol{v}\mid\boldsymbol{p}) \to \frac{1}{M}\boldsymbol{B}_p \operatorname{var}(\boldsymbol{f}\mid\boldsymbol{p})\boldsymbol{B}_p'$

Applying the formulas for updating normal random variables, we can write the conditional variance of $\boldsymbol{v}$ as:

$$\begin{aligned}\boldsymbol{\Sigma}_{v|p} &= \operatorname{var}(\boldsymbol{v}\mid\boldsymbol{f},\boldsymbol{p}) + \operatorname{cov}(\boldsymbol{v},\boldsymbol{f}\mid\boldsymbol{p})\operatorname{var}(\boldsymbol{f}\mid\boldsymbol{p})^{-1}\operatorname{cov}(\boldsymbol{f},\boldsymbol{v}\mid\boldsymbol{p})\\ &= \operatorname{var}(\boldsymbol{e}\mid\boldsymbol{f},\boldsymbol{p}) + \underbrace{\operatorname{cov}(\boldsymbol{v},\boldsymbol{f}\mid\boldsymbol{p})\operatorname{var}(\boldsymbol{f}\mid\boldsymbol{p})^{-1}}_{\boldsymbol{B}_p}\operatorname{var}(\boldsymbol{f}\mid\boldsymbol{p})\underbrace{\operatorname{var}(\boldsymbol{f}\mid\boldsymbol{p})^{-1}\operatorname{cov}(\boldsymbol{f},\boldsymbol{v}\mid\boldsymbol{p})}_{\boldsymbol{B}_p'},\end{aligned} \tag{A12}$$

where the second line follows from $\boldsymbol{v} = \boldsymbol{B}\boldsymbol{f} + \boldsymbol{e}$. Because $\boldsymbol{\Sigma}_e$ has bounded eigenvalues as $N \to \infty$, $\frac{1}{M}\operatorname{var}(\boldsymbol{e}\mid\boldsymbol{f},\boldsymbol{p}) \to 0$ as $N, M \to \infty$. The proof will follow from showing that $\boldsymbol{B}_p$ equals the expression given in the proposition. Applying the rules for updating normal random variables and using $\boldsymbol{s} = \bar{\boldsymbol{v}}_i - \bar{\boldsymbol{v}} - \boldsymbol{\Lambda}_i'(\boldsymbol{x} - \bar{\boldsymbol{x}})$ gives:

$$\operatorname{cov}(\boldsymbol{v},\boldsymbol{f}\mid\boldsymbol{p})=\operatorname{cov}(\boldsymbol{v},\boldsymbol{f}\mid\boldsymbol{s})=\operatorname{cov}(\boldsymbol{v},\boldsymbol{f})-\operatorname{cov}(\boldsymbol{v},\boldsymbol{s})\operatorname{var}(\boldsymbol{s})^{-1}\operatorname{cov}(\boldsymbol{s},\boldsymbol{f})$$
$$=\boldsymbol{B}-\boldsymbol{B}\operatorname{cov}(\boldsymbol{f},\boldsymbol{s})\operatorname{var}(\boldsymbol{s})^{-1}\operatorname{cov}(\boldsymbol{s},\boldsymbol{f})-\operatorname{cov}(\boldsymbol{e},\boldsymbol{s})\operatorname{var}(\boldsymbol{s})^{-1}\operatorname{cov}(\boldsymbol{s},\boldsymbol{f}),$$

$$\operatorname{var}(\boldsymbol{f}\mid\boldsymbol{p})=\operatorname{var}(\boldsymbol{f}\mid\boldsymbol{s})=\boldsymbol{I}-\operatorname{cov}(\boldsymbol{f},\boldsymbol{s})\operatorname{var}(\boldsymbol{s})^{-1}\operatorname{cov}(\boldsymbol{s},\boldsymbol{f}),$$

$$\begin{aligned}\boldsymbol{B}_p&=\operatorname{cov}(\boldsymbol{v},\boldsymbol{f}\mid\boldsymbol{p})\operatorname{var}(\boldsymbol{f}\mid\boldsymbol{p})^{-1}\\&=\boldsymbol{B}-\operatorname{cov}(\boldsymbol{e},\boldsymbol{s})\operatorname{var}(\boldsymbol{s})^{-1}\underbrace{\left(\operatorname{var}(\boldsymbol{s}\mid\boldsymbol{f})+\operatorname{cov}(\boldsymbol{s},\boldsymbol{f})\operatorname{cov}(\boldsymbol{f},\boldsymbol{s})\right)}_{\operatorname{var}(\boldsymbol{s})}\operatorname{var}(\boldsymbol{s}\mid\boldsymbol{f})^{-1}\operatorname{cov}(\boldsymbol{s},\boldsymbol{f})\\&-\boldsymbol{B}\operatorname{cov}(\boldsymbol{f},\boldsymbol{s})\operatorname{var}(\boldsymbol{s})^{-1}\underbrace{\left(\operatorname{var}(\boldsymbol{s}\mid\boldsymbol{f})-\operatorname{var}(\boldsymbol{s})+\operatorname{cov}(\boldsymbol{s},\boldsymbol{f})\operatorname{cov}(\boldsymbol{f},\boldsymbol{s})\right)}_{=\operatorname{var}(\boldsymbol{s}\mid\boldsymbol{f})-\operatorname{var}(\boldsymbol{s}\mid\boldsymbol{f})=\mathbf{O}}\operatorname{var}(\boldsymbol{s}\mid\boldsymbol{f})^{-1}\operatorname{cov}(\boldsymbol{s},\boldsymbol{f})\\&=\boldsymbol{B}-\operatorname{cov}(\boldsymbol{e},\boldsymbol{s})\operatorname{var}(\boldsymbol{s}\mid\boldsymbol{f})^{-1}\operatorname{cov}(\boldsymbol{s},\boldsymbol{f}).\end{aligned}\tag{A13}$$

Substituting from (A1) for $\boldsymbol{s}$ in (A13) gives the expression for $\boldsymbol{B}_p$ in the proposition.

Step 3: The vector of average aggregate uninformed trades has bounded elements

From (12), the average aggregate uninformed trades are $(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I}+\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Lambda}_i'\bar{\boldsymbol{x}}$. From (A3), the equation that defines $\boldsymbol{\Lambda}_i$ can be written as:

$$\tfrac{1}{M}A\boldsymbol{\Sigma}_{v|p}=\boldsymbol{\Lambda}_i-\tfrac{1}{M}\boldsymbol{\Lambda}_u'-\boldsymbol{\Sigma}_{\bar{v}_i}(\boldsymbol{\Sigma}_{\bar{v}_i}+\boldsymbol{\Sigma}_s)^{-1}(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i').\tag{A14}$$

From (A6), we can substitute $\boldsymbol{\Lambda}_i-\frac{1}{M}\boldsymbol{\Lambda}_u'=\frac{M-1}{M}\boldsymbol{\Lambda}_i\left(\frac{M-1}{M}\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i'\right)^{-1}(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')$ into (A14) and rearrange it to get:

$$\begin{aligned}&(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I}+\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Lambda}_i'\\&\quad=(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}(\tfrac{M-1}{M}\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}-\tfrac{1}{M}A(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I}+\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Sigma}_{v|p}(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}(\tfrac{M-1}{M}\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}.\end{aligned}\tag{A15}$$

Taking $N,M\to\infty$, the right-hand-side approaches:

$$\boldsymbol{I}-\tfrac{1}{M}A(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I}+\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Sigma}_{v|p}.\tag{A16}$$

The informed trader's second-order condition implies that $\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i'$ has strictly positive eigenvalues, which implies that $(\boldsymbol{\Lambda}_i+\boldsymbol{\Lambda}_i')^{-1}$ is bounded. The matrix $\frac{1}{M}\boldsymbol{\Sigma}_{v|p}$ is bounded if *N* grows no faster than *M*, which holds under our assumption that they grow at the same rate. If $\boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1}$ is unbounded, then $\boldsymbol{\Sigma}_s\boldsymbol{\Sigma}_{\bar{v}_i}^{-1}\to\mathbf{O}$. We now show that this would violate the informed trader's second-order condition. From (A14), we have:

$$\tfrac{1}{M} A\boldsymbol{\Sigma}_{v|p} = \boldsymbol{\Lambda}_i - \tfrac{1}{M}\boldsymbol{\Lambda}_u' - (\boldsymbol{I} + \boldsymbol{\Sigma}_s \boldsymbol{\Sigma}_{\bar{v}_i}^{-1})^{-1}(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i'). \tag{A17}$$

We have already established that $\tfrac{1}{M}\boldsymbol{\Lambda}_u' \to \mathbf{O}$ so that, if $\boldsymbol{\Sigma}_s \boldsymbol{\Sigma}_{\bar{v}_i}^{-1} \to \mathbf{O}$, then the right-hand-side of (A17) approaches $-\boldsymbol{\Lambda}_i'$. But this implies that $\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i' \to -2\tfrac{1}{M} A\boldsymbol{\Sigma}_{v|p}$, which is negative semi-definite and violates the informed trader's second-order condition. This implies that the matrix $(\boldsymbol{\Lambda}_i + \boldsymbol{\Lambda}_i')^{-1}(\boldsymbol{I} + \boldsymbol{\Sigma}_{\bar{v}_i}\boldsymbol{\Sigma}_s^{-1})\boldsymbol{\Lambda}_i'$ has bounded eigenvalues in the limit. Because $\bar{\boldsymbol{x}}$ has bounded elements, this implies that $M\,\mathrm{E}[\boldsymbol{d}_m]$ has bounded elements. In other words, uninformed traders do not take any unboundedly large positions in individual shares, on average.

Completing the proof

Step 1 and Step 3 imply that $\boldsymbol{\Lambda}_u'\,\mathrm{E}[\boldsymbol{d}_m] \to \mathbf{0}$. Because the elements of $M\,\mathrm{E}[\boldsymbol{d}_m]$ are bounded, they do not 'cancel out' the idiosyncratic risks that Step 2 shows approach zero in $\tfrac{1}{M}\boldsymbol{\Sigma}_{v|p}$ so that $A\boldsymbol{\Sigma}_{v|p}\,\mathrm{E}[\boldsymbol{d}_m] \to A\boldsymbol{B}_p \boldsymbol{\Sigma}_{f|p} \boldsymbol{B}_p'\,\mathrm{E}[\boldsymbol{d}_m]$, giving (22). ■

Proof of Proposition 3

Given the assumptions on $\boldsymbol{\Sigma}_{\bar{v}_i}$ and $\boldsymbol{\Sigma}_x$, we can pre- (post-) multiply (A4) by $\boldsymbol{T}'$ ($\boldsymbol{T}$) to and use $\boldsymbol{T}\boldsymbol{T}' = \boldsymbol{T}'\boldsymbol{T} = \boldsymbol{I}$ to obtain:

$$\begin{aligned} \mathbf{O} &= \tfrac{2(M-1)}{2M-1}\boldsymbol{\Theta}_\lambda \boldsymbol{\Theta}_x \boldsymbol{\Theta}_\lambda \boldsymbol{\Theta}_i^{-1}\boldsymbol{\Theta}_\lambda - A\tfrac{1}{M}\boldsymbol{\Theta}_\lambda \boldsymbol{\Theta}_x \boldsymbol{\Theta}_\lambda \boldsymbol{\Theta}_i^{-1}\boldsymbol{\Theta}_v - \tfrac{2M}{2M-1}\boldsymbol{\Theta}_\lambda - A\tfrac{1}{M}(\boldsymbol{\Theta}_v - \boldsymbol{\Theta}_i) \\ &\Rightarrow \mathbf{O} = \boldsymbol{\Theta}_x \boldsymbol{\Theta}_i^{-1}\boldsymbol{\Theta}_\lambda^3 - \tfrac{2M-1}{2(M-1)}A\tfrac{1}{M}\boldsymbol{\Theta}_x \boldsymbol{\Theta}_i^{-1}\boldsymbol{\Theta}_v \boldsymbol{\Theta}_\lambda^2 - \tfrac{M}{M-1}\boldsymbol{\Theta}_\lambda - \tfrac{2M-1}{2(M-1)}A\tfrac{1}{M}(\boldsymbol{\Theta}_v - \boldsymbol{\Theta}_i). \end{aligned} \tag{A18}$$

where we have used $\boldsymbol{T}'\boldsymbol{\Lambda}_u'\boldsymbol{T} = \tfrac{M}{2M-1}\boldsymbol{\Theta}_\lambda$ and the fact that diagonal matrices commute. Because the matrices in (A18) are diagonal, we can express it as N independent scalar equations:

$$0 = \tfrac{\theta_{kx}}{\theta_{ki}}\lambda_k^3 - \tfrac{2M-1}{2(M-1)}A\tfrac{\theta_{kx}}{M}\tfrac{\theta_k}{\theta_{ki}}\lambda_k^2 - \tfrac{M}{M-1}\lambda_k - \tfrac{2M-1}{2(M-1)}A\tfrac{1}{M}(\theta_k - \theta_{ki}), \tag{A19}$$

each of which is solved by a unique $\lambda_k > 0$ because only the leading λ_k^3 term has a positive coefficient. The cubic (A19) can be solved to give a closed-form solution for λ_k; however, we do not show the solution because the expression is long and not very useful.

Substituting from $\boldsymbol{\Sigma}_v = \boldsymbol{T}\boldsymbol{\Theta}_v\boldsymbol{T}'$, $\boldsymbol{\Sigma}_{\bar{v}_i} = \boldsymbol{T}\boldsymbol{\Theta}_i\boldsymbol{T}'$, $\boldsymbol{\Sigma}_x = \boldsymbol{T}\boldsymbol{\Theta}_x\boldsymbol{T}'$, and $\boldsymbol{\Lambda}_i = \boldsymbol{T}\boldsymbol{\Theta}_\lambda\boldsymbol{T}'$ into the expected returns (11) yields:

$$\mathrm{E}[\boldsymbol{v}-\boldsymbol{p}] = \tfrac{1}{2}\boldsymbol{T}\left(\tfrac{1}{2M-1}\boldsymbol{\Theta}_\lambda + A\tfrac{1}{M}\left(\boldsymbol{\Theta}_v - \boldsymbol{\Theta}_i^2(\boldsymbol{\Theta}_i + \boldsymbol{\Theta}_\lambda^2\boldsymbol{\Theta}_x)^{-1}\right)\right)(\boldsymbol{I} + \boldsymbol{\Theta}_i\boldsymbol{\Theta}_\lambda^{-2}\boldsymbol{\Theta}_x^{-1})\boldsymbol{T}'\bar{\boldsymbol{x}}. \tag{A20}$$

Because $\mathrm{E}[\boldsymbol{f}-\boldsymbol{p}_f] = \boldsymbol{\Theta}_v^{-1/2}\boldsymbol{T}'\,\mathrm{E}[\boldsymbol{v}-\boldsymbol{p}]$, (A20) implies (24). The second expression for $\mathrm{E}[f_k - p_{fk}]$ in (24) follows from rearranging (A19) to get:

$$\tfrac{1}{M}A\theta_{k|p} = \frac{2(M-1)}{2M-1}\frac{1-\frac{M}{M-1}\frac{\theta_{ki}}{\lambda_k^2\theta_{kx}}}{1+\frac{\theta_{ki}}{\lambda_k^2\theta_{kx}}}\lambda_k, \tag{A21}$$

and substituting into the first expression for $\mathrm{E}[f_k - p_{fk}]$ in (24). ■

Proof of Corollary 3.1

The term that multiplies $\bar{x}_{fk}$ in the first expression of $\mathrm{E}[f_k - p_{fk}]$ in (24), which, in conjunction with the second expression for $\mathrm{E}[f_k - p_{fk}]$, implies that $\lambda_k > \sqrt{\frac{\theta_{ki}}{\theta_{kx}}}$.

For the first part of the corollary, define the right-hand-side of (A19) as $g(\lambda_k)$. Because $g'(\lambda_k) > 0$ at the equilibrium λ_k, $\frac{\mathrm{d}\lambda_k}{\mathrm{d}\theta_{kx}} \propto -\frac{\partial g}{\partial\theta_{kx}}$. Direct computations give:

$$\frac{\partial g}{\partial\theta_{kx}} = \lambda_k^3\frac{1}{\theta_{ki}} - \frac{2M-1}{2(M-1)}A\frac{1}{M}\frac{\theta_k}{\theta_{ki}}\lambda_k^2 = \frac{1}{\theta_{kx}}\left(\frac{M}{M-1}\lambda_k + \frac{2M-1}{2(M-1)}A\frac{1}{M}(\theta_k - \theta_{ki})\right) > 0, \tag{A22}$$

where the second equality follows from a substitution from (A19). Noise trade variance impacts expected returns via the noise term $\lambda_k^2\theta_{kx}$ and the 'standalone' effect of λ_k in the $\frac{1}{2M-1}\lambda_k$ term in (24). This gives:

$$\frac{\mathrm{dE}[f_k - p_{fk}]}{\mathrm{d}\theta_{kx}} = \underbrace{\frac{\partial\,\mathrm{E}[f_k - p_{fk}]}{\partial\lambda_k^2\theta_{kx}}}_{<0}\underbrace{\frac{\mathrm{d}\lambda_k^2\theta_{kx}}{\mathrm{d}\theta_{kx}}}_{>0} + \frac{1}{2}\left(1 + \frac{\theta_{ki}}{\lambda_k^2\theta_{kx}}\right)\frac{1}{2M-1}\underbrace{\frac{\mathrm{d}\lambda_k}{\mathrm{d}\theta_{kx}}}_{<0} < 0. \tag{A23}$$

The inequality $\frac{\partial\,\mathrm{E}[f_k - p_{fk}]}{\partial\lambda_k^2\theta_{kx}} < 0$ follows from direct computations. We previously showed $\frac{\mathrm{d}\lambda_k}{\mathrm{d}\theta_{kx}} < 0$.

The inequality $\frac{\mathrm{d}\,\lambda_k^2\theta_{kx}}{\mathrm{d}\,\theta_{kx}} > 0$ follows from computations that give $\frac{\mathrm{d}\,\lambda_k^2\theta_{kx}}{\mathrm{d}\,\theta_{kx}} \propto \lambda_k^3 \frac{\theta_{kx}}{\theta_{ki}} - \frac{M}{M-1}\lambda_k > 0$, where the inequality follows from (A19). Because $\theta_{k|p}$ is increasing in $\lambda_k^2\theta_{kx}$, it is increasing in θ_{kx}. The change in the informed trade is:

$$\frac{\mathrm{d}\,y_{fk}}{\mathrm{d}\,\theta_{kx}} = \frac{1}{2}\frac{\theta_{ki}}{(\lambda_k^2\theta_{kx})^2}\,\overline{x}_{fk}\underbrace{\frac{\mathrm{d}\,\lambda_k^2\theta_{kx}}{\mathrm{d}\,\theta_{kx}}}_{>0} - \frac{1}{2\lambda_k^2}\,\overline{f}_{ki}\underbrace{\frac{\mathrm{d}\,\lambda_k}{\mathrm{d}\,\theta_{kx}}}_{<0}. \tag{A24}$$

Both terms in (A24) indicate the informed trader becomes more aggressive as noise trade variance increases. The actual trade size y_{fk} may decrease depending on the realization of $\overline{f}_{ki}$.

For the second part of the corollary, if $\lambda_k^2\theta_{kx} \to \infty$, then (A19) implies that $\lambda_k \to \frac{2M-1}{2(M-1)} A \frac{1}{M}\theta_k$. If $\lambda_k^2\theta_{kx} \to 0$, then (A19) implies that $\lambda_k \to -\frac{2M-1}{2M} A \frac{1}{M}(\theta_k - \theta_{ki}) < 0$, which violates the informed trader's second-order condition and implies that $\lambda_k^2\theta_{kx} \nrightarrow 0$. If $\theta_{kx} \to \infty$, then $\lambda_k^2\theta_{kx} \to \infty$, giving $\lambda_k \to \frac{2M-1}{2(M-1)} A \frac{1}{M}\theta_k$ and the statements for $\theta_{kx} \to \infty$. If $\theta_{kx} \to 0$, then λ_k becomes unbounded at a rate of $\theta_{kx}^{-1/2}$; otherwise, $\lambda_k^2\theta_{kx}$ approaches infinity, implying a bounded λ_k and $\lambda_k^2\theta_{kx} \to 0$, a contradiction, or $\lambda_k^2\theta_{kx} \to 0$, which generates a violation of the informed trader's second order condition. Expression (A19) can be rearranged as:

$$\lambda_k^2\theta_{kx} = \frac{\frac{M}{M-1}\lambda_k + \frac{2M-1}{2(M-1)}A\frac{1}{M}(\theta_k - \theta_{ki})}{\lambda_k - \frac{2M-1}{2(M-1)}A\frac{1}{M}\theta_k}\theta_{ki} \to \frac{M}{M-1}\theta_{ki} \quad \text{as } \lambda_k \to \infty. \tag{A25}$$

This gives the statements regarding $\theta_{kx} \to 0$, where the statement regarding returns stems from the $\frac{1}{2M-1}\lambda_k$ term that represents imperfect competition among uninformed investors in (24). ■

Proof of Corollary 3.2

If the monopolist trader has no private information $(\theta_{ki} = 0)$, then (A19) implies that

$\lambda_k = \frac{2M-1}{2(M-1)} A \frac{1}{M} \theta_k$, giving expected returns of $\mathrm{E}[f_k - p_{fk}] = \frac{1}{2}\frac{2M-1}{2(M-1)} A \frac{1}{M} \theta_k \bar{x}_{fk}$. Comparing to the expected return (24), we have:

$$\underbrace{\tfrac{1}{2}\tfrac{2M-1}{2(M-1)} A \tfrac{1}{M} \theta_k \left|\bar{x}_{fk}\right|}_{\text{Expected return with no private information}} < \underbrace{\left(\tfrac{1}{2M-1}\lambda_k + A\tfrac{1}{M}\theta_{k|p}\right)\tfrac{1}{2}\left(1+\tfrac{\theta_{ki}}{\lambda_k^2\theta_{kx}}\right)\left|\bar{x}_{fk}\right|}_{\text{Expected return with private information}} \tag{A26}$$

$$\Leftrightarrow \tfrac{1}{M-1}\lambda_k + \tfrac{2M-1}{2(M-1)} A \tfrac{\theta_k - \theta_{ki}}{M} > 0,$$

where the second line of (A26) follows after a substitution from (A19) and the inequality always holds because $\lambda_k > 0$, by the informed trader's second-order condition, and $\theta_k > \theta_{ki}$. ■

Proof of Proposition 4

The elements of the vector $\boldsymbol{x}_f = \boldsymbol{\Theta}_v^{-1/2}\boldsymbol{T}'\bar{\boldsymbol{x}}$ remain bounded. The eigenvector $\boldsymbol{t}_k$ of systematic risk $k \le K$ has elements of order $N^{-1/2}$ (See the proof of the Remark) so that the N-element sum $\boldsymbol{t}_k'\bar{\boldsymbol{x}} \sim N^{1/2}$ and $x_{fk} = \theta_k^{-1/2}\boldsymbol{t}_k'\bar{\boldsymbol{x}} \sim 1$ since $\theta_k^{-1/2} \sim N^{-1/2}$ for systematic risks.

For the systematic factors, $\theta_k, \theta_{ki} \sim N$. If $\theta_{kx} \sim N$, then the coefficients in (A19) are all bounded, implying that λ_k is bounded, as well. If $\theta_{kx} \sim N^{\alpha}$, $\alpha < 1$, then an argument similar to that used in proving Corollary 3.1 implies that $\lambda_k \sim N^{(1-\alpha)/2}$. In both cases, the term in expected returns (24) $\frac{1}{2M-1}\lambda_k \to 0$ because M increases faster than λ_k. In the latter case ($\theta_{kx} \sim N^{\alpha}, \alpha < 1$), rearranging (A19) and taking limits, using the fact that $\lambda_k \to \infty$, implies that $\lambda_k^2\theta_{kx} \to \theta_{ki}$. Substituting into expected returns (24) and informed trade (25) yields (26) and (27).

For the idiosyncratic factors, if θ_{kx} remains bounded, then taking limits on (A19) implies that $\lambda_k \to \sqrt{\frac{\theta_{ki}}{\theta_{kx}}}$. If θ_{kx} becomes unbounded, then (A19) implies that $\lambda_k \to 0$ and $\lambda_k^2\theta_{kx} \to \theta_{ki}$. These two facts give the second part of the proposition. ■

Proof of Corollary 4.1

We can write the expected returns as follows where the K subscript corresponds to the first K elements of the respective expression, corresponding to systematic risks, and the e subscript refers to elements $K+1, K+2, \ldots$, corresponding to idiosyncratic risks:

$$\mathrm{E}[\boldsymbol{v}-\boldsymbol{p}] = \boldsymbol{T}\boldsymbol{\Theta}_v^{-1/2}\,\mathrm{E}[\boldsymbol{f}-\boldsymbol{p}_f] = \boldsymbol{T}_K\boldsymbol{\Theta}_K^{-1/2}\,\mathrm{E}[\boldsymbol{f}_K-\boldsymbol{p}_{fK}] + \boldsymbol{T}_e\boldsymbol{\Theta}_e^{-1/2}\,\mathrm{E}[\boldsymbol{f}_e-\boldsymbol{p}_{fe}]. \tag{A27}$$

Proposition 4 shows that the elements of $\mathrm{E}[\boldsymbol{f}_e-\boldsymbol{p}_{fe}]$ approach zero, while the elements of $\boldsymbol{T}_e\boldsymbol{\Theta}_e^{-1/2}$ are bounded, giving the result of the corollary. ■